ANALYSIS OF COGGING TORQUE DUE TO MANUFACTURING VARIATIONS IN FRACTIONAL PITCH PERMANENT MAGNET SYNCHRONOUS MACHINES

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A thesis for the degree of Doctor of Philosophy at the School of Engineering and Information Technology, Charles Darwin University, Australia Submitted on the 12\textsuperscript{th} of November, 2013.
Declaration

I hereby declare that the work herein, now submitted as a thesis for the degree of Doctor of Philosophy at the Charles Darwin University, is the result of my own investigations, and all references to ideas and work of other researchers have been specifically acknowledged.

I hereby certify that the work embodied in this thesis has not already been accepted in substance for any degree, and is not being currently submitted in candidature for any other degree.

Mark Thiele
Abstract

Fractional pitch slot / pole arrangements are commonly used in permanent magnet synchronous motors (PMSMs) to significantly reduce cogging torque, however, maximum benefit depends on accurate stator and rotor manufacturing and alignment. This thesis presents a method of identifying the sources of manufacturing induced cogging torque.

Decoupling of cogging caused by stator and rotor manufacturing variation is possible due to stator and rotor affected harmonics being independent of one another. A hybrid FEA / analytical method was developed to simulate cogging torque with multiple manufacturing faults induced which proved to be 6 orders of magnitude faster than FEA alone. The new method utilises a library of FEA derived pole transition over single stator slot waveforms which are then assembled in the correct order and phased corresponding to the slot / pole interactions for a given PMSM.

Angular and eccentric misalignment can be identified by investigating the presence of first and second order sidebands about the pole and slot harmonics. Static angular misalignment induces first order sidebands around the slot harmonics while dynamic angular misalignment induces sidebands about the pole harmonics. Static eccentricity induces both first and second order sidebands around the slot harmonics, while dynamic eccentricity has the same effect on the pole harmonics.
A ‘Fault Diagnosis Flow Chart’ is presented to identify the manufacturing sources of additional harmonics. To demonstrate the effectiveness of the proposed method, all combinations of 10 production stators and 10 rotors of 24 slots and 10 poles in an axial flux configuration were experimentally measured and analysed. The stator slot variation and pole misplacement were found to be the largest contributors to unexpected cogging torque. Pole strength variation and static angular misalignment had minor contributions while dynamic angular, static eccentricity and dynamic eccentricity were found to not have a significant impact on the motors tested.
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Results from this research contributed to the following publications:


The work conducted as part of this thesis contributed to these publications:


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<td>PMSM</td>
<td>Permanent Magnet Synchronous Motor</td>
</tr>
<tr>
<td>PM</td>
<td>Permanent Magnet</td>
</tr>
<tr>
<td>AF</td>
<td>Axial Flux</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital Signal Processor</td>
</tr>
<tr>
<td>J&lt;sub&gt;s&lt;/sub&gt;</td>
<td>Stator angular moment of inertia</td>
</tr>
<tr>
<td>θ&lt;sub&gt;s&lt;/sub&gt;</td>
<td>Stator angular position</td>
</tr>
<tr>
<td>T&lt;sub&gt;motor&lt;/sub&gt;</td>
<td>Motor torque</td>
</tr>
<tr>
<td>T&lt;sub&gt;measured&lt;/sub&gt;</td>
<td>Measured torque</td>
</tr>
<tr>
<td>J&lt;sub&gt;r&lt;/sub&gt;</td>
<td>Rotor angular moment of inertia</td>
</tr>
<tr>
<td>θ&lt;sub&gt;r&lt;/sub&gt;</td>
<td>Rotor angular position</td>
</tr>
<tr>
<td>b&lt;sub&gt;bear&lt;/sub&gt;</td>
<td>Roller bearing viscous friction coefficient</td>
</tr>
<tr>
<td>θ&lt;sub&gt;B&lt;/sub&gt;</td>
<td>Brake (dynamometer) angular position</td>
</tr>
<tr>
<td>LCM</td>
<td>Lowest common multiple</td>
</tr>
<tr>
<td>N&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Number of poles</td>
</tr>
<tr>
<td>N&lt;sub&gt;s&lt;/sub&gt;</td>
<td>Number of stator slots</td>
</tr>
<tr>
<td>D/A</td>
<td>Digital to analogue converter</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>α&lt;sub&gt;skew&lt;/sub&gt;</td>
<td>the ratio of magnet circumferential skew to the slot pitch</td>
</tr>
<tr>
<td>EMF</td>
<td>Electromotive Force</td>
</tr>
<tr>
<td>bEMF</td>
<td>back Electromotive Force</td>
</tr>
<tr>
<td>Pole</td>
<td>One or more magnets forming a single magnetic pole. In this research, magnet and pole are used interchangeably as each pole consists of one magnet.</td>
</tr>
<tr>
<td>X</td>
<td>matrix of pole transition waveforms, each phased to a known position.</td>
</tr>
</tbody>
</table>
Glossary of Terms

$X^+$ the Moore-Penrose pseudo inverse of $X$

$\bar{y}$ least squares minimization vector of strengths

$i$ torque vector

$\bar{y}$ best linear combination of waveform magnitudes, or pole strengths, that explain the actual production rotor cogging torque.

$\bar{y}_{\text{average}}$ average strength of all poles.

$X_{\text{perfect}}$ matrix of perfect placement of poles

$F_s$ force in direction $s$

FEA finite element analysis

$W_1$ magnetic co-energy at position 1

$W_2$ magnetic co-energy at position 2

$\Delta S$ small displacement in the direction of the force

$K_{\text{total}}$ stiffness of the total system

$K_{\text{sensor}}$ sensor stiffness

$K_{\text{shaft}}$ shaft stiffness

$G$ modulus of torsional rigidity, which for structural steel is 80 GPa.

$D$ diameter of a solid shaft.

$L$ shaft length

$J$ angular moment of inertia

$A_{g_0}$ Nominal air gap

$A_{g_0}$ static air gap

$\alpha_s$ static angle to be simulated

$\theta_s$ angle of the position of the slot to be simulated

$\theta$ mechanical angle of rotation

$Ecc(\alpha)$ eccentricity at any angle $\alpha$

$\text{max}(Ecc)$ maximum eccentricity to be analysed

$A_d$ dynamic angle to be simulated

$\theta_p$ position of the pole to be simulated

$ID$ internal diameter of the rotor or stator

$OD$ external diameter of the rotor or stator

$\beta$ timing change angle induced by eccentricity
Chapter 1

Introduction

The use of permanent magnets (PMs) in synchronous motors offers significant advantages in terms of cost, power density, efficiency and control [1, 2]. It is for these reasons that their use and range of applications is expanding in spite of recent and significant fluctuations in the cost of the rare earth raw materials used in the manufacture of the magnets [3]. However, one significant disadvantage is cogging torque [4-6]. Cogging torque is caused by the unbalanced attraction of the permanent magnets to the ferromagnetic stator slots (or teeth) [7] and is present in nearly all types of high efficiency permanent magnet synchronous motors (PMSMs), also referred to as brushless DC motors [8]. Cogging torque is undesirable as it is a major source of vibration and noise [9], which in the worst cases can make PMSMs unsuitable for noise sensitive applications such as certain air conditioning system installations or situations requiring low speed operation with high rotational accuracy [4, 10].

Many documented methods are available to motor designers to reduce cogging torque [4, 5, 11-17], however, often their maximum potential benefit is not
achieved due to manufacturing induced variations. Thus, in reality, most PMSMs exhibit significantly more cogging torque than they theoretically should [18, 19].

This introduction will present background information on the various types of PMSMs including an overview of sources of noise and vibration. Cogging torque minimisation techniques are presented, with a justification for the research. The proposed method for solving the problem is outlined and finally a chapter overview is presented.

1.1 Background

PMSMs can offer substantial advantages over other motor types and configurations, however excessive cogging torque can lead to noise and vibration potentially rendering these motors unsuitable for certain noise or vibration sensitive applications. The existing body of knowledge has identified key causes of additional cogging torque [4, 5, 10, 11, 16, 18, 20-23], however, what is not as well quantified is the percentage of the total cogging torque that can be attributed to each of the different sources [24].

Additionally, methods of predicting cogging torque have been proposed [25-32] and these can model perfect PMSM configurations however, they are unable to be used to model non-ideal motor configurations such as magnet misplacement, magnet strength variation or slot placement either in combination or individually. These manufacturing induced imperfections, along with various misalignments of the rotor and stator could all be found with mass produced machines. The stochastic nature of the possible manufacturing problems results in FEA methods being an inelegant solution, or if a range of variables are to be determined, unfeasible from a calculation time perspective with individual 3D solutions taking potentially hundreds of hours to solve [33].
1.2 Motor Types

Industrial electric drives can be categorised into two basic types: direct current (DC) and alternating current (AC) machines, with AC being further classified into asynchronous or synchronous.

- DC drives were the first developed and are the simplest to control, however, they require more regular maintenance and are less reliable than AC drives due to the need for brushes which are a consumable [34].
- AC asynchronous drives (also called induction machines or drives) are the most common industrial drive due to their low cost, simple construction and reliability, however speed control is difficult and they are less efficient when compared to AC synchronous and DC drives [35, 36].
- AC synchronous drives, sometimes referred to as brushless DC, offer similar performance to DC drives with superior reliability as the motors do not have brushes [34]. They can be further subdivided:
  - Switched Reluctance – have no permanent magnets but rather rely on reluctance of the rotor ‘poles’ being attracted to the stator electromagnetic poles to generate torque [37].
  - Synchronous Reluctance – similar to switched reluctance they generally have no permanent magnets. The rotor has varying circumferential permeance induced by flux barriers to concentrate stator induced flux to particular regions in the rotor to generate torque [38, 39].
  - Radial flux PM– poles are circumferentially mounted on the rotor with the stator radially positioned around the perimeter as shown by the cross section in Figure 1.1. The poles can be either surface mounted on the rotor as in the figure or imbedded in the rotor body, referred to as surface inset when the PMs are level with the surface or interior permanent magnet (IPM) if the PMs are well below the surface [40, 41].
1 Introduction

- Axial flux PM – flat poles are mounted on a rotor disk facing the stator as shown in Figure 1.2. Axial flux configuration motors have considerable volumetric and therefore mass and cost advantages as well as faster acceleration [40] than radial configuration PMSMs, as well as AC synchronous motors of the same torque output [42].

For both radial and axial configurations, surface PMs could also take the form of a magnet continuum, also referred to as a ring magnet, where a ‘ring’ of PM material is magnetised with discrete areas of north and south poles and the ring is slid over (radial) and / or bonded to (axial) the rotor. However, as continuous PMs are more expensive than individual magnets [40] the focus of this research will remain with individual surface mounted PMs. Additionally, given the potential performance advantages offered by axial configuration PMSMs along with the unique challenges with respect to their manufacture, it is these types of machine that will be the subject of this research.
Figure 1.1 Radial flux topology with surface mount poles, coils not shown for clarity.
1.3 Cogging Torque

In most situations, fluctuation in output torque is undesirable. Deviation from constant torque output is called pulsating torque and it has two main sources: cogging torque and torque ripple [43].
1 Introduction

**Cogging Torque**

The design of most PMSM stators consists of copper wire wound around laminated steel cores to concentrate the magnetic flux and improve torque density. The poles on the rotor are strongly attracted to the individual steel cores and torque is required to `cog’ the poles from one set of cores to the next. Thus, cogging torque is the interaction of the rotors’ magnetic flux with variations in the stators magnetic reluctance \[43\]. Cogging torque is always present irrespective of whether the PMSM is rotating or stationary or is driving or being driven, that is, unpowered. It is considered the more difficult source of pulsating torque to compensate for [44] and thus there has been less success in providing reliable and manufacturing tolerance insensitive methods of eliminating it. Cogging torque can be up to 3% of rated motor torque according to Grcar et. al. [44], however, others have suggested a nominal value of between 5 – 10% of rated torque [40], although in poorly designed PMSMs it may be as high as 25%.

**Torque Ripple**

Torque ripple is caused by the stator current magnetomotive forces interacting with the rotors magnetic flux distribution [43, 44]. Torque ripple occurs only when the machine is powered and is influenced by both motor design and the motor controller or control methods [45], and as such a mismatch between current shape and back EMF can be solved by changing the motor design or the current shape. While torque ripple can be between 2 and 4% of rated torque, it is the subject of recent research [45-60] in PMSMs, much of which has proposed solutions that reduce torque ripple to less than 1% of rated torque [61]. Additionally, for the production PMSMs at the focus of this research, cogging torque was the more significant issue and the back EMF variation due to manufacturing variations is a subject for further investigation and therefore is not the focus of this work.
1.4 Sources of Cogging Torque and Methods of Reduction

1.4.1 Sources of Cogging Torque

Cogging torque sources can be grouped into two broad categories [18]:

1. Design sources
2. Manufacturing induced sources

Design sources of cogging torque are those that are inherent in the PMSM design and are fully expected in the final product, while manufacturing induced cogging torque results from the imperfections associated with mass production, such as the misplacement of poles and variations in magnet strengths. It is these manufacturing induced cogging torque components that lead to additional cogging torque harmonics that are not expected in the design of the PMSM which are also difficult to identify and will form the focus of this research.

1.4.2 Methods of Reducing Cogging Torque

There are two general methods for reducing cogging torque in PMSMs. These are mechanical methods and control based methods.

1.4.2.1 Mechanical Methods

Numerous mechanical methods have been documented for the reduction of cogging torque which directly address the root cause [4]. As cogging torque stems from the interaction of the poles with the ferromagnetic stator components, resolution of the problem at this level is independent of any other
system and is often the most effective means of minimising cogging torque [43, 44]. Furthermore, identification of the root cause may allow resolution of the problem without any additional cost or complexity.

The commercially available PMSMs that were the basis of this research utilised fractional pitch and magnet skew to reduce cogging torque, however, there is a discrepancy between the theoretical or maximum effectiveness of these mechanisms and the production motors actual cogging torque. It is known that manufacturing induced variations of motor components can significantly increase cogging torque [10, 14, 18, 27] and some research has identified key contributors or critical components [4, 43]. However, a quantification or sensitivity analysis on the manufacturing processes and their relationship to cogging torque has not been published. That is, the contribution of each of the key motor components to the overall PMSM cogging torque has to date not been accurately quantified.

1.4.2.2 Control Methods

Controller based torque ripple minimisation is achieved by optimising the interaction between the phase currents and the phase back EMF (bEMF) waveforms. The control of the phase currents can be altered to cancel cogging torque to provide smoother torque output, however, there are two main problems associated with this solution. In Jahns and Soong [43] it was suggested that successful implementation of control methods to eliminate cogging torque depends on either “accurate turning” or predetermination of the motor characteristics and that success requires the use of an encoder. To date there is limited research published which presents a control method that will compensate for manufacturing induced cogging torque without accurate position information from an encoder, one of the few being from Saunders et. al. [62]. Cost prohibits the installation of encoders in most industrial applications. Additionally, adaptive control methods do not address the underlying root cause
of the problem and it was concluded by Jahns and Soong [43] that it is generally preferable to solve pulsating torque problems at the motor design level rather than with adaptive control strategies. However, recent advances in PMSM control, including sensorless control, may be able to compensate for some or a majority of the cogging torque [63-65], nonetheless as designers and manufacturers strive to build quieter motors, they will implement the most economical method of eliminating or reducing motor noise and these decisions can only be made if all the options are fully understood. As such this research will focus on the mechanical causes of cogging torque and therefore the mechanical methods which may be suitable for reducing cogging torque.

1.5 Documented Methods of Analysing Cogging Torque

Cogging torque can be analysed by various methods, the suitability of each will depend on several factors which will be briefly introduced.

Analysis methods can be divided into two general categories, experimental and theoretical based. The advantages of experimental data when measured with an accurate system relate to the certainty that the acquired data is a true reflection of the test motor’s cogging torque, however, it is generally not feasible to experimentally assess many motor configurations as part of a design process. Theoretical methods offer the advantage of being able to predict cogging torque without the need for a prototype motor.

Finite element analysis is now widely employed in the analysis of cogging torque [12, 66-78] and can provide accurate results [7]. However, as axial flux machines are inherently 3D, the subsequent FEA models require significant computer resources and time to solve, thereby limiting their suitability for multi degree of freedom system analysis and optimisation.
An alternative to FEA is to utilise analytical methods [29, 71, 75, 79-84] which are orders of magnitude faster and therefore require significantly less computer resources, however, their ability to accurately model and account for all possible manufacturing induced imperfections is limited and this has led to the development of hybrid FEA and analytical methods [33].

Several of the documented analytical and hybrid methods rely on superposition, which, in relation to cogging torque requires that a motor's complete cogging torque can be constructed from the sum of its parts and that standard underlying superposition assumptions hold true [19, 85-87].

Hybrid analytical / FEA methods have been developed which aim to achieve a compromise between the accuracy of FEA with the speed of analytical methods [33, 88, 89]. Typically they use FEA to simulate a small portion of the motor as the source and then use analytical methods to reconstruct complete waveforms of the parameter of interest.

### 1.6 Proposed Method for Analysing Cogging Torque

The proposed method for determining the causes of cogging torque consists of accurate cogging torque measurement, conducting harmonic analysis on the waveforms and using superposition to reconstruct a complete motor’s cogging torque from a library of FEA derived waveforms constructed from two magnet poles passing an individual stator slot. Harmonic analysis techniques evaluate repetitive components or periodic signals buried in noisy vibration signals from rotating machinery.
1.7 Research Goal

This research will present and validate a method for determining the manufacturing parameters responsible for additional cogging torque in production motors, providing motor manufacturers and designers with a tool to allow cogging torque reduction in a cost effective manner.

1.8 Research Approach

Completion of the research goal will involve completing the following tasks:

1. Literature review of:
   a. Published causes of cogging torque
   b. Effects of manufacturing variations on cogging torque
   c. Methods for predicting and analysing cogging torque
   d. Methods for measuring cogging torque
2. Implementation and comparison of existing methods for decoupling sources of cogging torque.
3. Creation of a new hybrid FEA and analytical method for the prediction of cogging torque due to manufacturing induced errors.
4. Designing an experimental setup to accurately measure cogging torque on production motors.
5. Analysing the experimental cogging torque data to determine individual motor parameter contributions to cogging torque

1.9 Chapter Overview

To present the outcomes of the above task list, chapter 2 will review the published material relating to cogging torque, its causes, how it is measured and
1 Introduction

the methods for reduction. Chapter 3 reviews the methods suitable for analysing cogging torque and their application while chapter 4 introduces how the cogging torque was measured experimentally and the data obtained. Chapter 5 then presents the implemented methods for decoupling manufacturing causes of cogging torque and chapter 6 presents and discusses the results of the testing and analysis. The implications of the research for manufactures are presented in chapter 7 and conclusions are made in chapter 8, with recommendations as well as suggestions for motor manufacturers and designers and areas for further research.
Chapter 2

Review – Cogging Torque

The introduction highlighted the importance of minimising cogging torque in PMSMs and focused the research on analysing the motor mechanical parameters responsible for creation of cogging torque in a mass produced axial flux PMSM. The initial section of this chapter reviews published techniques and strategies for motor designers and manufacturers to reduce levels of cogging torque, how they work and their effectiveness. The effect of manufacturing induced motor variations on cogging torque is then considered and finally, options for measuring cogging torque experimentally are examined.

2.1 Design Sources of Cogging Torque and Mechanical Minimisation Methods

The power density, efficiency and cost advantages offered by PMSMs, and particularly axial flux PMSMs is leading to an expanded range of applications, some of which have strict noise and vibration requirements such as electric power steering systems [14, 71, 90-93] and industrial and automotive air
Cogging Torque

Cogging is a source of vibration which can lead to undesirable acoustic noise and adversely affect motor performance when precise rotational positioning is required [9].

These stricter requirements are driving PMSM designers and manufacturers to implement multiple strategies in order to minimise cogging torque and therefore the noise and vibration. These strategies will now be discussed, with emphasis on those widely implemented in industry.

Cogging torque minimisation strategies exist that can be applied to the stator or the rotor and these are summarised in Figure 2.1 [11].

Figure 2.1. Summary of cogging torque minimisation strategies for PMSMs [11].

2.1.1 Fractional Pitch

Fractional pitch utilises a fractional number of rotor poles to stator slots while integral pitch motors have an integer number of poles to stator slots [4, 34].

Several papers have identified the slot to pole number as being one of the most critical aspects of motor design for the effective minimisation of cogging torque...
2 Review – Cogging Torque

[4, 10, 47, 95, 96]. This is due to the partial cancellation of cogging detent forces which has the effect of greatly reducing the magnitude and increasing the frequency of cogging torque harmonics. The partial cancellation of cogging components occurs due to pairs of poles being attracted to stator teeth in opposite directions, shown in Figure 2.2.

Figure 2.3 shows an integral setup where all the poles align to stator teeth concurrently and the cogging torque is the cumulative torque required to index all the poles from one detent position to the next.

![Fractional pitch PMSM, radial flux shown without windings for clarity. $N_p = 10$ & $N_s = 12$ in this case.](image)
Figure 2.3 Integral pitch PMSM, radial flux shown without windings for clarity. Note $N_p = N_s = 12$ in this case.

Figure 2.2 represents a fractional pitch arrangement where the number of poles fully aligned to stator teeth is greatly reduced when compared to the integral pitch configuration. The actual number of poles aligning to stator teeth depends on the fractional pitch arrangement, with that shown in Figure 2.2 representing a 10 pole / 12 slot arrangement. Vector cancellation of the PM attractive forces greatly reduces the cogging torque magnitude while increasing its frequency.

While minimised cogging torque is generally considered advantageous, there may be situations when a high cogging torque could be beneficial such as assisting with zero speed holding torque. While an integral pitch motor would offer this advantage there are also related disadvantages. In addition to the much larger cogging torque, integral pitch PMSMs are wound with distributed windings where the copper coils are distributed over multiple slots [40]. This winding layout increases the volume of copper required and therefore increases copper losses, reduces efficiency and influences the shape of the back EMF (bEMF) waveform [34, 97]. Fractional pitch machines are wound with concentrated windings where the copper coils are wound around only one stator
tooth [40] and have a more sinusoidal bEMF and reduced end turns resulting in a reduction in copper losses and the mass of copper required for the windings [97]. It is for these reasons in addition to the cogging torque benefits that fractional pitch machines are generally preferred over integral pitch.

As well as overall motor efficiency, selection of the pole to slot ratio will result in changes to the cogging torque levels. In Zhu and Howe [10], the authors present a ‘cogging torque goodness factor’ for any pole / slot combination, which calculates the maximum number of aligned poles and slots at any time and is equal to the multiple of the number of poles and slots divided by the lowest common multiple. I.e.

\[ C_T = \frac{N_p \cdot N_s}{LCM} \]  \hspace{1cm} \text{Eq 1} \\

Where:

- \( N_p \) is the number of poles
- \( N_s \) is the number of slots and
- \( LCM \) is the lowest common multiple.

It is commonly accepted that a slot pole combination with a high least common multiple (LCM) results in the lowest cogging torque at the highest frequency [4, 10, 96], however, the choice of slot and pole number combination will also depend on other constraining design criteria such as slot width, tooth width and magnet pole arc / pole pitch ratio [96]. It should also be noted that the cogging torque will not reduce to zero. The cogging torque remaining from the poles that do not cancel, referred to as a Native Harmonic Component [18, 20] will always exist, even in perfectly manufactured motors. For fractional pitch to provide the maximum reduction in cogging torque, there is the requirement of accurate and precise manufacturing to ensure the poles are correctly placed,
their strengths are uniform and the stator teeth have uniform magnetic reluctance and accurate placement. Manufacturing induced variations of these characteristics will result in the attractive forces not being cancelled completely, the consequence being higher than expected cogging torque. Any cogging torque that results from imprecision within the motor from normal mass production variations is referred to as Additional Harmonic Components of cogging torque [18, 20] and is reviewed in further detail in section 2.1.7.1 below.

2.1.2 Magnet Skew

Cogging torque is caused by the PM interacting with the varying stator magnetic reluctance. An unskewed pole interacts with the stator slot along its entire radial edge resulting in abrupt changes in torque which in turn creates a cogging torque profile with a large magnitude. If the magnet shape is modified, for example, by using disk shaped magnets in an axial flux machine, then the magnet interaction with the stator tooth is also modified and the duration of engagement extended. This ‘softer’ interaction of the magnet with the tooth reduces the magnitude of the cogging torque profile as shown in work conducted by Aydin [11]. He found that increasing levels of magnet skew resulted in reductions of the cogging torque magnitude when analysing different options with 3D FEA and comparing these results to a reference motor. He concluded that significant reductions in cogging torque were possible with magnet skewing.

Theoretically, with the magnet skewed one full slot pitch, as is common, the cogging torque should become zero, however in practice this does not occur due to magnetic flux leakage at the inner and outer radii of the poles [11]. Therefore, the minimum cogging torque will rarely be zero and the optimum skew angle may not be one full slot pitch. Further, magnet skewing only reduces the higher order native harmonic components of cogging torque and will not affect cogging torque components produced by manufacturing imperfections [11].
Although one full slot pitch magnet skewing is common practice \cite{11, 96}, there may be motor configurations where less than one full slot pitch is also effective. Depending on the number of poles and slots, skew angles that are a fraction of a full slot pitch could be equally effective. It was found by Zhu and Howe \cite{10} that there was equal effectiveness of different skew angles as long as the following held true:

\[
\frac{\alpha_{\text{skew}} \cdot LCM}{N_s} = \text{any integer}
\]

Eq 2

where:

\( \alpha_{\text{skew}} \) = the ratio of circumferential skew to the slot pitch,

\( LCM \) = the lowest common multiple between the number of poles and the number of slots

\( N_s \) = the number of slots on the stator.

For the PMSM investigated in this research, the possible skew ratios were determined as follows:

\[
\alpha_{\text{skew}} = \frac{k \cdot N_s}{LCM} \quad k = 1, 2, \ldots, \frac{LCM}{N_s}
\]

Eq 3

For the PMSM investigated, \( N_s = 24 \) and \( LCM = 120 \), therefore,

\[
\alpha_{\text{skew}} = \frac{24k}{120} \quad k = 1, 2, 3, 4, 5.
\]

Eq 4

Therefore, \( \alpha_{\text{skew}} \) could be 0.2, 0.4, 0.6, 0.8 or 1, that is, a full slot pitch, however, skewing to the minimum effective slot pitch should maximise efficiency by minimising leakage inductance and copper losses \cite{11}.
From a cogging torque perspective, there should be little difference between the result of skewing the poles or the stator slots, however, only limited research has been found that discusses the benefits or detriments of skewing the stator slots [10, 95], with the identified possible negative consequences being that it may lead to an increase in the leakage inductance and the copper losses which would therefore decrease motor efficiency. Additionally, manufacturing a skewed stator would be more complex, particularly for an axial flux PMSM. Given the relative simplicity of manufacturing skewed poles for axial flux motors, this will always be the preferred method.

Hwang et al. [95] conducted a comparison of skewed and unskewed radial flux stators with a predicted reduction in cogging torque of at least 25% with the introduction of magnet skew in three motor combinations. However, the amount of skew is not discussed, nor are the unskewed results measured; 2 dimensional FEA is used to predict unskewed cogging torque with good correlation between FEA and measured data for the skewed setup. Additionally, the paper does not state how the skewed cogging torque was experimentally measured (down to 0.02 Nm) or any aspect of the measurement apparatus.

2.1.3 Dummy or Auxiliary Slots

Dummy slots are additional and non-functional slots that can be added to stator teeth (Figure 2.4) resulting in an increase in the frequency and a reduction in the magnitude of cogging torque. Bianchi and Bolognani [5] added two dummy slots to each stator tooth effectively increasing the number of teeth by three and compared the cogging torque before and after the addition using FEA. The cogging torque frequency increased by a factor of three while the magnitude reduced by the same amount with the introduction of the dummy slots on a 24 pole 18 slot radial flux PMSM. The disadvantages associated with the use of dummy slots are an increase in the manufacturing complexity of the stator and
an increase in the stator reluctance and therefore a reduction in PMSM efficiency and mean torque output.

![Dummy slots](image)

**Figure 2.4 Example of a stator with one dummy or auxiliary slot per tooth.**

Dummy slots may offer manufacturers some ability to ‘tune’ the cogging torque frequency for particular applications. For example, it may allow fundamental frequencies of surrounding structures to be avoided thereby preventing resonance and acoustic noise. It may also increase the frequency of the cogging torque to above the audible range.

### 2.1.4 Pole Arc to Pole Pitch Ratio

Figure 2.5 illustrates pole arc width and pole pitch for an axial flux PMSM. The number and width of the poles on a rotor is an important parameter in reducing cogging torque [4, 5, 10, 12, 16]. However, this can be difficult to design as the optimum ratio also depends on the air gap, the magnetisation pattern of the
PMs and the width of the stator slots. Additionally, small variations in the PM pole arc can result in significant changes to the cogging torque [5]. Supporting and emphasising this point, the effect of varying pole arc to pole pitch ratios on a 9 slot / 8 pole, a 6 slot / 4 pole and a 12 slot / 4 pole configuration motors were investigated by Zhu and Howe [10]. Using a 2D analytical method, they found that as pole arc to pole pitch ratio was increased, the cogging torque initially reduced then passed through an optimum value after which the phase was reversed and the magnitudes increased.

Figure 2.5 Axial rotor illustrating the definitions of pole arc width and pole pitch.

2.1.5 Variable Pole Arcs

It is possible to have poles of differing face widths or pole arcs to reduce the cogging torque in a similar mode to fractional pitch [4, 5, 10], however there are several drawbacks. Firstly, poles of different sizes are required which increases the cost of magnet tooling and therefore the cost of the magnets as well as the
complexity of pole assembly on the manufacturing line. It also affects the back EMF waveform making the stator design more complex. For these reasons it is normally not implemented by motor designers [5].

2.1.6 Stator Slot Opening

Another strategy for reducing cogging torque from the stator side is to modify the opening of the stator slot and several options are investigated in Bianchi and Bolognani [5] and Islam, Mir et. al. [14], where the slot width is reduced (Figure 2.6).

![Figure 2.6 Axial stator shown with modified slots (from CAD).](image)

Cogging torque is reduced due to the narrowing of the stator slot and the smoother transition of the pole from one stator tooth to the next, however, one drawback is that the narrower slots mean fewer windings can be inserted and the winding process is more difficult. An alternate method is to use separate ‘slot close overs’ where a thin piece of additional material covers the entire stator slot. This has been shown to reduce cogging torque, although additional noise may result if the close overs are able to vibrate against the stator [43, 98].
2.1.7 Manufacturing Variations and Their Effect on Cogging Torque

Many of the methods discussed in section 2.1 are capable of reducing cogging torque, however, their effectiveness also depends on the accuracy with which the motor can be manufactured. For example, fractional pitch is highly effective in reducing cogging torque however, its full potential is only realised when the following conditions are met –

- All poles have identical magnetic material configuration and flux density with alternating magnetisation direction
- All poles are placed in the perfect position
- All slots are in the perfect position
- Magnetic reluctance of the stator material is uniform
- Alignment of the rotor to the stator is accurate

If these conditions are met, then the cogging torque generated is called the ‘Native Harmonic Component’ (NHC) of cogging torque [10, 14, 18] and is the ideal. It relates to the number of pole / slot interactions for each revolution of the motor and is less than the product of the pole and slot number as some interactions occur simultaneously. It can be predicted using FEA or analytical methods and is directly related to the lowest common multiple of the slot and pole number. Thus, the native harmonic component will have orders of:

\[ NHC = LCM\left(N_s, N_p\right) \cdot i \quad i = 1, 2, 3 \ldots \]  \hspace{1cm} \text{Eq 5}

where:

- \( NHC \) = orders of Native Harmonic Component
- \( LCM \) = lowest common multiple
- \( N_s \) = number of slots
In reality, the above ideal conditions are rarely possible, particularly in a mass produced PMSM. With the introduction of manufacturing variations such as inaccurate pole placement, so too are ‘Additional Harmonic Components’ of cogging torque [18], and the total cogging torque is the sum of the native (ideal) and additional (manufacturing induced) harmonics. This is discussed in further detail in section 2.1.7.1 below.

Numerous papers have been published documenting methods and strategies for the reduction of cogging torque in permanent magnet machines as presented in section 2.1, however, few papers have been published which accurately quantify the contribution of manufacturing inaccuracies with specific motor parts to unexpected cogging torque. Islam et. al. [14] assessed the effect of a range of motor parameters on cogging torque on a 27 slot 6 pole radial flux PMSM. FEA and analytical data are validated with experimental results to determine the PMSM cogging torque sensitivity to variations in:

- Magnet Shape and Magnetization Pattern
- Magnet Dimension and Placement Irregularities
- Slot Opening in the Stator Lamination
- Stator Lamination Anisotropies
- Soft Magnetic Material in the Motor Housing
- Rotor Eccentricity and Air-Gap Variation
- Magnet Pole Arc Design
- Dummy Slots
- Skewing of Rotor Magnets
- Step Skewing

The experimental motor’s cogging torque is compared with four types of magnet skewing implemented. The unskewed condition peak to peak (p-p) cogging was approximately 0.03 Nm. This reduced to approximately 0.01 Nm when magnet
skew equal to one full slot pitch was implemented. Due to the manufacturing complexity associated with skewing radial magnets, two variations of step skewing were also implemented, one step and two step skewed poles. With step skewing, poles are constructed from smaller straight arc magnets which are shifted to cancel different harmonic components in the total cogging torque. The experimental data suggested that the p-p cogging reduced to approximately 0.015 Nm with one step and 0.012 Nm with two steps. However, the harmonic content of the two was also different, with the one step skew reducing the 27th harmonic and the two step skew reducing both the 27th and 54th harmonics.

Interestingly, some of the simulated situations are not fully representative of a mass produced motor with manufacturing induced variations. As an example, only the north poles of a 6 pole 27 slot PMSM are shifted from their ideal position which showed that increasing misplacement resulted in an increase in peak to peak cogging torque and concludes ‘It can be seen that the accuracy in magnet placement is critical for minimizing the cogging torque.’. However, the increase in cogging torque resulting from the misplacement is not representative of a production motor, where pole misplacement would more likely be of a random normally distributed nature and affect both north and south poles. Additionally, there is no quantification of the percentage reduction in cogging torque possible with improvements in pole placement or how it compares to other motor parameters. Similarly, Gašparin *et. al.* [27] and Cernigoj *et. al.* [20] investigate several motor characteristics such as pole position, thickness and width and their effects on cogging torque. They measure the cogging torque and the pole position on 20 production rotors to conclude that “…analysis carried out on 20 PM rotors presents that cogging torque is mostly effected by improper PM positions on rotors...”. However, there is no discussion of how the cogging torque was measured, the accuracy of either the pole position measurements nor of the potential improvements possible with perfect placement.
Coenen et. al., [99] considered the design of a PMSM to reduce its sensitivity to magnet flux distribution and static eccentricity of the rotor due to manufacturing tolerances. The research also investigates the influence of pole width and slot width on cogging torque using FEA. The findings suggest that a motor design can be optimised to limit the influence of manufacturing imperfections on cogging torque allowing for a potential widening of tolerances on certain parts and therefore an associated reduction in part cost. Alternatively, the existing tolerances can be maintained and an improvement in cogging realised. The FEA and analytical data are not validated with experimental work and there is no reference to the type of PMSM investigated, be this radial or axial flux, the pole or slot count. Therefore, while reductions in cogging torque of 10% are presented, it is not clear what the actual cogging torque figures are as a percentage of rated torque. There is the possibility that large reductions are possible from machines with relatively high levels of initial cogging torque which may not be effective on machines with already low levels of cogging.

In another study, Ombach and Junak [92] analysed three manufacturing processes for an 8 pole 12 slot radial flux motor both in terms of their variation in production and their effect on cogging torque. Using 3D FEA, they analysed three motor configurations as shown in Table 1.

**Table 1. Motor configurations analysed by Ombach and Junak [92]**

<table>
<thead>
<tr>
<th>Motor Characteristic</th>
<th>Configuration 1</th>
<th>Configuration 2</th>
<th>Configuration 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skew (-ve skew is backward swept)</td>
<td>0°</td>
<td>-0.5°</td>
<td>0.5°</td>
</tr>
<tr>
<td>Pole misplacement</td>
<td>0 mm</td>
<td>-0.2 mm</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>Pole strength deviation between N &amp; S poles</td>
<td>0%</td>
<td>-0.5%</td>
<td>0.5%</td>
</tr>
<tr>
<td>Cogging torque</td>
<td>0.5 Ncm</td>
<td>1.9 Ncm</td>
<td>1.8 Ncm</td>
</tr>
</tbody>
</table>
Table 1 shows that the two motor configurations with manufacturing induced variations (configurations 2 & 3) have increased cogging torque by a factor of between 3 and 4. However, the degree to which each of the three production variables individually contributed to the increases in predicted cogging torque is not quantified. A sensitivity analysis on the three production variables would provide useful information to designers and manufacturers as there is the possibility that one source is responsible for a majority of the increase in cogging while the others have limited effect.

2.1.7.1 Native and Additional Harmonic Components

Native and additional harmonic components of a cogging torque waveform were briefly introduced in section 2.1.7 above. This section will review the separate contributors to additional harmonic components.

Additional harmonic components arise due to manufacturing imperfections with the rotor, stator and assembly; however, each of the different sources contributes a unique harmonic. Gašparin et. al. [18, 27] suggested that stator affected frequency harmonics of the cogging spectrum are multiples of the number of poles and rotor affected harmonics are multiples of the number of slots.

The research conducted by Gašparin et. al. [18] found that linearly increasing manufacturing tolerances or more severe manufacturing induced imperfections, such as pole misplacement, resulted in a linear increase in the additional harmonic components associated with the particular imperfection while the native harmonic components (those due to the motor design) are negligibly affected. The significance in these results is that it presents a possible means of identifying the individual contributors to the total cogging torque due to
manufacturing tolerances. The data was sourced from a 6 pole, 27 slot radial flux motor in which pole assembly tolerances of +0.3°, 0° and -0.3° misplaced from the ideal 60° were simulated (that is one of three possible positions for each of the 6 poles or 729 different assembly combinations) and showed a strong correlation between the additional harmonic components and the severity of the misplacement. Pole thickness and width were also modelled with similar results. One minor limitation of the research relates to the fact that the real pole assembly situation would be a normal random distribution within, for example, the range of ±0.3° and more data points may show a non-linear relationship between pole placement and additional harmonic magnitude. Another relates to each of the possible manufacturing assembly inaccuracies being dealt with independently of each other. As an example, the research describes the effect of pole misplacement and pole thickness variations on the cogging torque additional harmonic components separately, but how the two interact when combined is not considered.

One of the important outcomes of the research conducted by Gašparin et. al. [18] is that it clearly relates rotor affected harmonics to the number of slots and stator affected harmonics to the number of poles presenting a possible means of decoupling stator and rotor contributions to cogging torque.

While the above research correlates cogging torque to pole placement, width and thickness accuracy, others have also shown the same is true for stator assembly accuracy [100]. Some motors use modular stators where individual stator teeth are wound and then assembled into a complete stator, with the aim of improving manufacturability and packing factor of the windings. However, one downside is that there exists the possibility of additional gaps between stator segments which also induces added components capable of non-uniformity or randomness which results in the incomplete cancellation of cogging forces. This was investigated by Zhu, et. al. [100] on a 10 pole, 12 slot fractional pitch interior PM radial configuration motor designed for an
automotive electric power steering system. The stator was of a modular design with both segmented radial teeth and circumferential yoke, thereby the possibility of gaps between any of these segments was deemed to be possible. The analysis predominantly used 2D FEA to predict the effect of uniform and non-uniform gaps between modules with the results suggesting that uniform gaps increase the cogging torque magnitude but do not affect its periodicity while non-uniform gaps affected both. The modular stator with only one stator tooth having an additional 0.01 mm air gap was found to have a significantly increased cogging torque. Theoretically, according to FEA, the perfectly assembled PMSM should have had peak to peak cogging torque in the range of ±100 mNm, however, with one stator module misplaced by 0.01 mm, the peak to peak cogging torque increase to ±700 mNm, thereby showing the sensitivity of stator accuracy to cogging torque.

The cogging torque of a production machine was experimentally measured and its cogging torque found to be similar to that of a FEA model having non-uniform stator gaps (one at 0.06 mm with the remainder at 0.025 mm), however, no geometric measurements were conducted to validate the presence (or otherwise) of stator gaps in the production motor and therefore the alignment of cogging torque waveforms from experimental and FEA analysis seems to offer a weak validation of the true root cause of the production machines additional cogging torque components.

In another study, Zhu et. al. [101] considers the impact of stator asymmetry caused by keybar slots, notches and flats added to the outside of a 6 pole, 36 slot (6p 36s) radial flux laminated stator. The sections around the asymmetries saturate causing a significant increase to cogging torque magnitude. Also interesting is the fact that the harmonic of the number of poles, \( N_p \), in the frequency spectrum is greatly increased with the introduction of the asymmetries, which correlates well with the research and findings conducted by Gašparin et. al. [18] as presented earlier.
2.1.7.2 Misalignment Factors Affecting Additional Harmonic Components.

Section 2.1.7.1 presented that manufacturing precision determines the additional harmonic content and design determines the native harmonic content. This section will review the alignment factors that lead to additional harmonic content in the cogging torque and identify a possible means of decoupling the separate sources.

The existing published research focusing on PMSM misalignment does not use consistent terminology for describing misalignment faults, particularly between radial and axial flux PMSMs. For example, the static eccentricity fault that Jagasics [102] describes in a radial flux PMSM is different to the static eccentricity fault described by Mirimani et al. [103] in an axial flux arrangement. In order to avoid confusion with various definitions used to describe misalignment, the definitions used in this research will now be presented.

2.1.7.3 Static Angular Misalignment Definition

Static angular misalignment in an axial flux PMSM is caused by the stator not being parallel to the rotor, either due to problematic stator mounting or the rotor shaft axis being non perpendicular to the face of the stator due to the bearing housing being incorrectly bored. It is referred to as static as the rotor spins truly but is angled to the stator. It can be diagrammatically represented as shown by Figure 2.7.

Pure static angular misalignment results in each slot (static reference frame) having a constant but unique air gap while the air gap for each pole (rotating reference frame) varies sinusoidally with each revolution of the rotor.
2.1.7.4 Dynamic Angular Misalignment Definition

In this research, dynamic angular misalignment in an axial flux PMSM is caused by the stator not being parallel to the rotor either due to a bent rotor shaft or the rotor shaft bore being non-perpendicular to the rotor face. It is referred to as dynamic as the rotor appears to ‘wobble’ around the stator. It can be diagrammatically represented as in Figure 2.8.

For pure dynamic angular misalignment, each individual pole on the rotor (rotating reference frame) has a constant but unique air gap whereas each slot (static reference frame) sees a sinusoidal variation of air gap for each rotor revolution.
2.1.7.5 Static Eccentricity Misalignment Definition

For this research, the definition of static eccentricity is applied geometrically to the axial flux PMSM. Therefore static eccentricity occurs when the stator and rotor are offset from one another but the rotor is spinning about its own true centre, as shown by the schematic representation in Figure 2.9, and as would occur if, for example, a stator was offset from the rotational axis of the rotor or the rotor bearing bore was incorrectly positioned. It results in variations in eccentricity for poles as they rotate, from maximum overhung of the OD to maximum overhung of the ID 180° mechanical later. From a fixed reference frame, each of the slots sees a unique but constant eccentricity, while from a rotating reference frame, the eccentricity varies sinusoidally.
Figure 2.9 Exaggerated schematic representation of static eccentricity misalignment. Note that the rotor shaft is offset from the stator but central in the rotor, i.e., the rotor is spinning about its own true centre which is offset from the stators geometric centre.
2.1.7.6 Dynamic Eccentricity Misalignment Definition

Dynamic eccentricity occurs when the rotor shaft is offset from the rotor centre, as may be caused by the incorrect positioning of the rotor bore, and shown in the diagrammatic representation in Figure 2.10. From a rotating reference frame, the poles have a constant eccentricity while from a static reference frame the slots have a sinusoidal variation.

Figure 2.10 Schematic representation of dynamic eccentricity caused by the rotor shaft bore being offset from the true centre. The rotor shaft is central and spins about the true centre.

2.1.7.7 Misalignment Review of Published Work

The research conducted by le Roux et al. [104, 105] investigated the effect of static and dynamic eccentricity and pole strength variation on stator current and voltage of a 4 pole 36 slot radial flux PMSM. The research used 2D FEA to predict flux linkages and inductances for a complete revolution, sampling every 0.36° (1000 steps per revolution) to effectively create a lookup table for four machine
configurations. Specifically, a perfectly constructed motor (aligned and without faults), and motors with 32% static eccentricity, 22% dynamic eccentricity and finally a motor with one pole 22% smaller to simulate a missing pole piece. Simulation data was compared to experimental data for the same faults with good agreement. The results showed that dynamic eccentricity and the magnetic flux variation could be detected in the fault frequencies of the PMSM, but the static eccentricity was not detectable as there was no change in air gap associated with the rotating rotor. While no reference is made to cogging torque, it does highlight that static eccentricity may be difficult to detect through means other than cogging torque measurement. Interestingly, and without explanation, le Roux et al. found that the amplitude of the fault frequencies for configurations where static eccentricity was induced were lower than the aligned conditions. Perhaps this was due to a reduction of the flux linkage due to the eccentricity. Finally, the researchers proposed a means of dynamic eccentricity fault detection by identifying the magnitude of the 0.5th fault harmonic for the motor investigated.

Jagasics [102] published research on the relationship between static eccentricity and pole / slot ratio combinations which showed that some arrangements were more sensitive to this type of manufacturing fault than others. For example, the FEA analysed 10 pole, 12 slot configuration was relatively insensitive to 0.2 and 0.4 mm of eccentricity (20% and 40%) however, a 10 pole 9 slot configuration had a threefold increase in its cogging torque with the same increase in eccentricity, although this may be due in part to the much lower initial cogging which was approximately 8 times smaller than the 10p / 12s PMSM.

The analysis was conducted using only FEA and no experimental validation was presented. Jagasics refers to mesh distortion issues that result from transient analysis and can lead to errors. This was overcome by adding an additional layer between the air gap to increase the mesh density in this critical region which is also re-meshed for every time step, minimising mesh distortion and inaccuracy.
While it does not consider the effect on cogging torque, the work by Ebrahimi and Faiz [106, 107] considers the effect of static, dynamic and mixed eccentricity on the stator current of a radial flux PMSM using time stepping FEA which is validated with experimental data. The eccentricities in the test motor are introduced by machining both the bearing housing bore and the motor shaft so that wedges can be inserted to offset the rotor from the stator. A wedge placed between the bearing outer race and the housing induces static eccentricity while a wedge between the bearing inner race and the rotor shaft induce dynamic eccentricity. The researchers conducted spectrum analysis on stator currents and investigated the additional harmonic components induced by the faults. The findings indicated that the sideband components of the fundamentals were a good indicator of a fault and could be used for fault recognition in production motors. This highlights a potential means of detecting the effect of misalignment faults on cogging torque.

The earlier work by Ebrahimi et. al. [108] is similar however only considers static eccentricity rather than dynamic and mixed.

Mirimani et. al. [103] investigate what they referred to as static eccentricity, however, by the definitions outlined in section 2.1.7.2, would be referred to as static angular misalignment in a 28 pole, 24 slot axial flux PMSM by analysing the back EMF of four individual coils placed at 90° intervals. In an axial flux machine, static angular misalignment arises when the rotor is non-parallel to the stator and the position of the misalignment angle does not shift with rotor rotation, such as would be created by manufacturing inaccuracy of the angle of the rotor bearing bore or angular misalignment of the stator. The static eccentricity Mirimani et. al. refer to is derived from a radial configuration machine which, from a magnetic perspective, the induced faults are analogous. Geometric eccentricity in axial flux machines occurs when the rotor and stator centres are offset. As discussed earlier in this chapter, geometric descriptions of the misalignment faults are used in this research.
As would be expected, they found that the back EMF voltage increased from the coil with the reduced air gap, the coil with the increased air gap had a reduced bEMF voltage and the two coils at the positions where air gap was unchanged had no change to their bEMF. They used 3D FEA to predict the voltages at 10, 20, 30 and 40% static angular misalignment and validated the FEA data with experimental data at 40% static angular misalignment, with good correlation.

Recent work by Zhu et. al. [109] used analytical and FEA methods to assess the sensitivity of radial flux PMSMs with several slot / pole combinations to static and dynamic eccentricity. The six motors analysed were a 8 pole / 9 slot, 10 pole / 9 slot, 4 pole / 6 slot, 8 pole / 6 slot, 8 pole / 12 slot and a 4 pole / 12 slot. The analytical results were partially validated by experimentally testing both 9 slot machines. There were two significant findings from the research. One, PMSMs with similar pole / slot counts are more sensitive to eccentricity misalignment than those with larger differences, therefore, the 8 pole / 9 slot and 10 pole / 9 slot machine were more significantly affected that the other motor combinations with the integral pitch machines being the least affected. Secondly, Zhu et. al. found that static eccentricity misalignment induced sidebands about the harmonic of the number of slots and dynamic eccentricity induced sidebands about the harmonic of the number of poles, confirming earlier publications by Ebrahimi and Faiz [106] and Thiele and Heins [110], the latter of which is covered in Section 5.6.

2.2 Cogging Torque Measurement Review

Analytical and FEA methods of cogging torque prediction are powerful and useful in certain situations, however, validation with experimental data is desirable for high levels of confidence. While many research papers do this, very few discuss the detail of the measurement method or system. This section will review documented methods and systems for the measurement of cogging torque.
PMSMs convert electrical to mechanical energy and vice versa and this conversion occurs in the air gap between the rotor PMs and the stator electromagnets, therefore this is the area of interest for the analysis of cogging torque. Owen et al. [111] propose three methods of pulsating torque measurement for induction motors:

1. Measuring instantaneous power consumption at the stator terminals
2. Mounting of accelerometers on the rotor to measure instantaneous angular accelerations, which, if the motor is uncoupled, would predominantly be due to pulsating torque, however, torque variations due to bearing friction, although usually small, cannot be excluded.
3. Measure air gap flux and stator current to develop a relation to air gap torque

Pulsating torque in an induction motor is caused by the stator current magnetomotive forces interacting with the rotor’s magnetic flux distribution, however, in PMSMs pulsating torque is the combination of both this and cogging torque, so the above methods 1 and 3 are not suitable. Mounting accelerometers on an externally driven rotor may be possible with the use of slip rings or possibly wireless communication, however, these alternatives are more complex than using a dedicated torque transducer and are an indirect method of measurement.

2.2.1 Sensor Selection

The conventional methods of torque measurement utilise one of the following sensors, each of which have advantages and disadvantages:

- Inline shaft torque sensor – a coupling mounted between the driver and the driven components which measures torsional shaft deflection using a strain gauge.
Advantages: a calibrated, reliable method of torque measurement within specified sensor ranges.

Disadvantages: as shaft torsional deflection is required, low system stiffness and therefore low resonant frequencies will limit the measurement range and the sample rate. If coupled to the rotating elements, mass inertia attenuation of the excitation forces may reduce the measurement system's ability to record pulsating torque or the measurement speed may need to be greatly reduced. Another consideration will be the resonant or eigen frequency for the rotor sub assembly which may interfere with the measurement frequency.

- Strain force gauge to measure reaction torque – a strain gauge measures the reaction load on either the driver or the driven components to determine torque [112].
  - Advantages: simple and reliable
  - Disadvantages: as deflection of some part of the system is required, low system stiffness and therefore low resonant frequencies will limit the measurement range and the sample rate.

- Piezoelectric reaction torque sensor – piezoelectric force sensors are used to determine torque reaction.
  - Advantages: low distortion and high stiffness. Low crosstalk (i.e. low influence from non-torque forces on the torque measurements) [113].
  - Disadvantages: high cost, only AC transient measurements possible with average or DC torque being unreliable. In reality, quasistatic (short term - less than 1 minute) DC measurements are possible. Measurements in excess of this time could result in a drift of the reading and a resultant reduction in measurement accuracy.
Caricchi *et. al.* [6], used a HBM torque meter rated to 1000 Nm with 0.1% precision coupled between the DC drive motor and an AFPM machine, however, there is the concern of torque ripple from the DC machine interfering with the measurements, as any torque ripple from the DC machine would adversely affect the cogging torque data, as was the case with research conducted by Qian *et. al.* [114].

Zhu [112] presented a simple and cheap method for measuring cogging torque in PM machines using a lever, a set of digital scales and a lathe to rotate the rotor to measure the cogging torque. The agreement between measurements and FE analysis is good. The measured cogging torque was conducted on a 2 pole, 2 slot machine, therefore the lowest common multiple is also 2 which means that a complete waveform is 180° of shaft rotation. This implies that a relatively low resolution of the shaft position is required. Although it is not discussed, it is envisaged that the measurement frequency would be very low and therefore not practical for measuring the cogging torque on many motor combinations to a high resolution, particularly if high order data is required. Having said that, if a limited number of experimental motor’s cogging torques are required to validate FEA or analytical data, the equipment could be found in most workshops or laboratories and therefore presents an inventive method of easily obtaining data. Additionally, the apparatus appears to have sufficient torque measurement resolution to measure the 0.13 Nm cogging torque of the 2 pole, 2 slot motor which is similar in magnitude to the motor at the focus of this research.

Sun *et. al.* [115] identify the problem of measuring a small AC torque component, such as torque ripple, with a large DC torque from the motor. They overcome this issue by using an inline torque sensor coupled to a flywheel so that only the AC component is present and a higher resolution torque transducer can be used.
2.2.2 Rotor Drive Method

In section 1.3 pulsating torque was defined as the sum of cogging torque and torque ripple, where the ripple component was electromagnetically induced from the operation of the motor and is only present when the motor is powered. Cogging torque exists whenever the motor is spinning thus to decouple cogging torque and torque ripple the rotor must be externally driven by a separate source to ensure that only the cogging torque component of the pulsating torque is measured. It is important that the external driving of the rotor does not in itself become a source of torque ripple as this would cause interference with the measured data.

The documented drive options include:

- External motor direct drive
- External motor indirect drive, i.e. belt
- Internal drive, i.e. the motor being investigated is driven by itself

Of these options, only those utilising an external drive motor meet the previously mentioned criteria of not inducing pulsating torque in addition to the cogging torque. However, selection of a suitable external drive is particularly important if an inline torque sensor were to be used on the rotating shaft as the rotors’ inertia would result in corruption of the data in line with the torque pulsations of the external drive. This was found by Qian et. al. [114] where the DC motor used as a load source interfered with the test motor pulsating torque.

2.2.3 Bearing Friction

Bearing or shunt loads [116] can either be a viscous torque from the lubricant or a higher frequency disturbance if imperfections exist in the bearing. The
presence of shunt loads in the measured torque depends on the relative location of the sensor and bearings.

The effect of bearing friction loads is further discussed in section 4.2.3.

### 2.2.4 Resonant Frequencies

Resonances can affect the frequency measurement range by amplifying or attenuating cogging torque before it is measured. To ensure measurement linearity error is less than 5%, all frequencies measured should be at least five times lower than the fundamental resonant frequency of the system [117] (p166).

Torque sensor bandwidth is an important consideration, however the more stringent requirement is for the bandwidth of the total mechanical system, of which the sensor is one component. The measurement system should be designed to have the highest practical fundamental resonant frequency and the operating range of the motor adjusted to ensure all harmonics of interest fall within the resulting accurate measurement range [31].

Designing an experimental test rig for optimised resonant frequencies from the ground up was performed by Heins, Thiele and Brown [118]. This process commenced with the optimisation of the most critical component from a resonant frequency perspective, the drive shaft in the case considered in the paper, and then progressed by designing the remaining components around it. This process ensured a maximisation of resonant frequencies and therefore provided the largest possible system bandwidth.
2.3 Summary – Cogging Torque Review

This chapter reviewed the literature on cogging torque minimisation methods and the effect of manufacturing variations on cogging torque where the concept of additional harmonic components was introduced. Finally, the experimental measurement of cogging torque was reviewed with the findings suggesting a piezoelectric reaction torque sensor and the rotor being indirectly driven by an external drive source appear to offer an acceptable solution.
Chapter 3

Review – Superposition, Order Analysis and FEA

Chapter 2 reviewed cogging torque minimisation methods, the measurement of cogging torque and the effect that manufacturing induced motor imperfections have on cogging torque. This chapter will review the non-experimental methods of analysing cogging torque, which are analytical methods, superposition and FEA.

Computationally efficient analytical methods are tools useful to motor designers and researchers as cogging torque can be predicted without significant investments of time in FEA models or prototype motors. Therefore, the effect of changes to particular motor parameters on cogging torque can be predicted quickly, allowing for rapid cogging torque optimisation for proposed design changes. However, as identified in chapter 2, a significant proportion of cogging torque results from manufacturing induced variations, these methods also need to be able to decouple the motor parameters that influence cogging torque, that is, slot position, pole position and pole strength and rotor and stator alignments.
Superposition in conjunction with analytical methods can be used to achieve the required decoupling [33] and the documented options will now be reviewed.

### 3.1 Superposition

Superposition has been successfully used to create or analyse cogging torque waveforms for PMSMs and is based on the theory that a complete waveform is the sum of all its parts [19, 30, 86, 87]. Therefore, 20 poles interacting with 24 slots (the motor used in this investigation) can be recreated from the interaction of a single pole transition with one slot which is then replicated, indexed and summed appropriately. This method presents a potential means of decoupling slot position, pole placement and pole strength as was demonstrated by Zhu *et. al.* [7, 86, 87], Liu *et. al.* [30] and also Heins, Brown and Thiele [119] where good correlation between cogging torque predicted by finite element methods, synthesised by superposition and measured data was obtained [7] [p8] [30] [p1224], [22].

According to Zhu *et. al.* [86], however, there are limitations which require quantifying prior to synthesis of highly accurate data. One such limitation is the effect of magnetic saturation which may result from the relative difference in flux density between a fully loaded rotor and a rotor partially loaded with poles. In some circumstances there could be an increase in the relative flux density with a partially loaded rotor if the backing plate was magnetically saturated in the fully loaded configuration. Conversely, if the rotor was loaded with only one magnet then there is no flux return path and flux density is reduced, leading to reduced accuracy. These situations induce a non-linearity into the system which would require verification prior to use of superposition, that is, quantification of the relative magnitudes of a reduced number of poles and / or slots compared to the magnitudes of the same as part of a rotor fully loaded with poles and a complete stator assembly.
In the same study, Zhu et al. [86] compared the cogging torque from a radial flux PMSM with 4 poles and 6 slots using three methods:

- Experimental measurement
- FEA
- Superposition of a single pole waveform

The findings indicated that the use of superposition resulted in significant errors if only one pole was used due to the elimination of the flux return path. However, when pairs of poles were used and the flux return paths closed, the difference between FEA and superposition predicted cogging torque was reduced to an average of 3.5%, however this was only for perfect motor configurations and did not include any manufacturing induced inaccuracies. The authors start with a Fourier series for a single pole passing $N_s$ number of slots, but this is not representative of what would be measured with just one pole due to the error associated with saturation. A second pole spaced $2\pi/N_p$ from the first pole, where $N_p$ is the number of poles, is added to the Fourier series and a two pole cogging torque determined. A relationship between one pole and two pole cogging torque is then obtainable. Thus, a cogging torque waveform from a fictitious single pole can be determined if a two pole waveform is obtained, either through analytical methods, FEA or experimentally. This ‘fictitious’ single pole has the same air gap field distribution as when it is paired, but can now be used for determining relationships between cogging torque and the influence of different slot and pole number combinations.

Zhu et al. also developed a similar but alternative approach for the analytical construction of cogging torque waveforms using a single stator slot and superposition [7] rather than the aforementioned single pole. Both these methods highlight that they can be used successfully to investigate the effect of motor design parameters such as fractional and integral pitch combination and that with the use of a fictitious single pole to overcome the effect of magnetic
saturation there is good agreement between FEA, analytical and experimental results. However, these methods are not used to develop analytical tools to predict the effect of manufacturing variations, such as pole misplacement, on cogging torque. That is, superposition was used to simulate only perfectly manufactured PMSM and these were compared to prototype experimental configurations, which may well be assembled to a higher degree of accuracy. Additionally, the FEA and experimental data are sourced on radial flux motor configurations. While it is not expected that the presented methods are specific to radial flux topologies, there is a significant difference between that published and this research which may require tailoring to axial flux topologies.

An extension of the research by Zhu et al. [31] was conducted by Liu and Li [120] with specific attention given to the pole transition over slot to further improve the existing analytical models capabilities, particularly with respect to the prediction of cogging torque. They found that a pole transition was able to replicate the rapid change in tangential flux density that occurs when poles traverse a single stator slot. The presented analytical solution accurately accounts for the pole transition passing the slot and predicts the flux density vector distribution. These distributions are then used to calculate the force by using the Maxwell stress tensor. The analytical solution was validated with FEA.

### 3.2 Order Analysis

Also referred to as harmonic analysis, order analysis techniques evaluate repetitive components or periodic signals buried in noisy vibration signals from rotating machinery. An order is defined as a harmonic of the rotational speed [121] and is applicable in this situation due to the periodic nature of the cogging torque signal. Order analysis enables dominant excitation forces to be extracted from an otherwise unobvious signal through the use of Fast Fourier Transform (FFT) algorithms. As discussed in section 2.1.7.1, a cogging torque waveform can
have multiple causes each of which can be attributed to a different source, specifically, rotor causes of cogging torque contribute to the $N_r$ harmonics and stator causes the $N_p$ harmonics, as was identified by Gašparin et. al. [18].

Gašparin et. al. tested 20 production PM motors from a radial flux 6 pole, 27 slot PMSM and determined the FFT of the cogging torque harmonics corresponding with the number of slots. The position of the poles was also measured to determine the relationship between the two parameters. They found there was a near linear relationship between the degree of pole misplacement and the magnitude in the harmonics corresponding with the number of slots, that is, the greater the misplacement of the poles, the larger was the magnitude of the 27th order harmonics from the FFT of the cogging torque. Additionally, Gašparin found a similar relationship between pole thickness and the additional cogging torque harmonics, presumably as this influences relative pole strength.

### 3.3 Analytical Solutions

For a motor designer, it will not always be possible to determine the cogging torque through measurement, particularly with new motor designs. For these reasons the ability to predict cogging torque is beneficial and can be achieved in several ways. The two main methods are to use FEA packages such as Ansys® Maxwell with either a 2D or 3D model, or an analytical approach to predict the cogging torque. Analytical solutions provide an alternate solution path requiring orders of magnitude less computation time [33] while still offering reliable results although they are generally considered less accurate than FEA [31, 75, 79, 80, 120, 122]. However, most analytical methods do not cater for manufacturing inaccuracies and they also assume a symmetrical cogging torque waveform, that is, the waveform in the first half of the period will be the inverse of that in the second half. While this may well be the case most of the time, it is possible non-
symmetrical pole shapes, such as when skew is implemented, may well result in asymmetric cogging torque waveforms.

Di Gerlando et al. [33], created analytical methods to predict the effect of dynamic angular misalignment (referred to in their research as manufacturing dissymmetry) on the axial and bending forces and stress, flux linkages, parallel loop circulating currents and also the bEMF waveform shape of a twin rotor, double sided stator axial flux PMSM. The developed analytical models agreed with FEA evaluated data and offered the advantage of greatly reduced computation time, with the 3D FEA taking over 214 hours compared to the analytical methods 2.2 minutes. However, the research assumed a geometrically perfect stator, ideal pole placement and coaxial alignment of the rotor to the stator, thereby limiting the manufacturing errors to dynamic angular misalignment and non-symmetrical air gap only. Also, only individual modules are considered rather than the entire motor, as would be required if further manufacturing errors were to be considered. The method used 2D FEA to acquire a permanent magnet field function and separately, a stator slot field function for a range of air gaps. As these 2D simulations do not account for radial end effects, an end effect modifier was developed for the air gaps simulated. These air gap modified field functions were then cubic spline interpolated over the entire range to create a lookup table for which seven motor configurations were simulated. The air gaps were determined for 4 levels of dynamic angle at 4 positions around the circumference. They found that increasing dissymmetry resulted in increasing forces on the stator and larger bending torques on the rotor shaft, in line with expectation. The research conducted by Di Gerlando et al. highlights the potential of obtaining a range of FEA data widely spaced and then post processing these data to create a finer data set to save on computation time. These data can then in turn be used to more accurately model manufacturing errors for linear systems.
Not all analytical models can accurately predict the desired motor parameter. For example, Zarko et. al. [122] presented an analytical method for the prediction of cogging torque where the peak predicted values were approximately double that predicted by FEA, that is 100% error. Although the method was able to accurately predict the effect of changing motor parameters on cogging torque, it also highlighted one of the limitations of such methods where accurate cogging torque data is required, possibly further emphasising the need for good experimental and / or FEA data.

Of the research published, most prefer to validate the analytical models developed with FEA while a few were validated with experimental data. An exception is the work conducted by Simón-Sempere et. al. [123] who develop an analytical statistical method to analyse the effect on the PMSM bEMF with variation in pole strength, position, width and angular displacement. The analytical data were validated by both numerical and experimental data, with four groups of motors forming the experimental pool. Of these four motor groups, one consisted of 35 preproduction prototype 100 W, 4 pole PMSMs and another had 35 production versions of the same motor. The third consisted of 8, 250 W, 8 pole motors and the forth group consisted of 8 180 W 6 pole versions, giving a total of 86 experimental motors. All were externally driven and the bEMF measured with a 14 bit data acquisition system. Their analysis was able to determine that the real and imaginary components of the harmonic voltages corresponded to different sources of error, specifically, the real was real due to pole position error and the imaginary component due to pole strength and width variation. Additionally, they were able to use their analysis method to assess the sensitivity of two possible motor designs to production variation and thereby select the most robust design.
3.4 Electromagnetic FEA

This section presents details of FEA software used in this research.

Finite element analysis (FEA) software is a widely used engineering tool used to predict physical outcomes involving properties such as fluid flow, thermal transfer, structural and electromagnetic interactions. The finite element method finds the solution to any problem that has a finite set of partial differential equations and can be used in static, steady state and transient situations. The solution of any finite element model is dependent on the size of the finite elements with the outcome trending towards an exact solution with decreasing distance between elements. The Ansys Maxwell electromagnetic FEA software utilised in this research uses Maxwell’s equations to solve the electromagnetic field problem within a defined region.

3.4.1 Maxwell’s Equations

There are four equations which are commonly referred to as Maxwell’s Equations, two of which describe how a field emanates from a source (Gauss’s Laws for Magnetism and Electricity) and two describe how it circulates (Faraday’s Law of Induction and Ampere’s Law).

The differential forms of the equations are [3, 124]:

Gauss’ Law for Electric Charge Density

\[ \nabla \cdot D = \rho \]  \hspace{0.5cm} \text{Eq 6} 

Faraday’s Law,
\[ \nabla \times E = -\frac{\partial B}{\partial t} \quad \text{Eq 7} \]

Gauss’ Law for Magnetism

\[ \nabla \cdot B = 0 \quad \text{Eq 8} \]

Ampere’s Law

\[ \nabla \times H = J + \frac{\partial D}{\partial t} \quad \text{Eq 9} \]

where:

- \(E\) is the electric field,
- \(D\) is the electric displacement, \(eE\),
- \(B\) is the magnetic flux density,
- \(H\) is the magnetic field intensity, \(B/\mu\).
- \(J\) is the conduction current density, \(sE\).
- \(\rho_v\) is the charge density.

### 3.4.2 Adaptive Mesh Analysis.

One of the most critical requirements to ensure reliable and accurate FEA results is to develop an appropriate mesh. For a mesh to be suitable, it must be fine enough to provide accurate and meaningful results and yet not so fine that computation time is excessive [124].

Ansys Maxwell has the capability to automatically generate an adaptive mesh based on certain criteria from the initial seed. An iterative process follows
whereby areas of significant error are refined until the minimum error criteria are met. This then allows relatively unimportant areas to be coarsely meshed with large tetrahedra while more important areas, such as pole interfaces, are meshed much more finely (Figure 3.1) thereby optimising the accuracy of the results obtained for the minimum computational time.

![Diagram showing adaptive meshing](image)

**Figure 3.1.** Example of FEA adaptive meshing showing mesh refinement in critical areas such as the stator tooth tips and magnets and a coarse mesh in less critical areas such as the stator yoke.

### 3.4.3 Convergence Study

Adaptive meshing is an applicable method for mesh creation for magnetostatic applications or where uniform magnetic flux density exists for the entire transient analysis, such as for a complete motor assembly with small gaps between poles. However, when a single or a pair of poles is to be analysed transiently, adaptive meshing is not suitable as mesh refinement only occurs where the initial flux density is high and decaying with a step gradient. As soon as a transient analysis is commenced and the position of the poles shifted, the high flux density region (and therefore high mesh density area) moves into a
region of course mesh and the accuracy of the model is compromised. For these reasons, manual mesh seeding and refinement is required, or transient simulation avoided in favour of multiple magnetostatic simulations with adaptive meshing every time step.

### 3.4.3.1 Relationship Between Mesh Density, Accuracy and Processing Time

The widely accepted method of determining if a mesh is acceptable is to continually halve the distance between mesh nodes until the output is no longer affected, that is, the point where increasing the mesh density does not significantly improve the accuracy of the model [30].

It should also be noted, however, that this method needs to be applied to the measurement of interest rather than a general property such as flux density or magnetic vector potential, for example. The reason lies in the fact that convergence for primary quantities is faster than for secondary or tertiary quantities such as force and torque. As an example, Salon [124] considered the force applied to an iron bar by a simple electromagnetic circuit. He found that the flux density was unchanged when the mesh was increased from 456 nodes to 700 nodes, but that the force did not converge until the mesh contained approximately 2000 nodes. That is, the mesh required for force convergence was three times more dense than that required for flux density convergence. This highlights the importance of determining mesh convergence based on the measurement of interest rather than the energy error or that if the energy error in the mesh is used as the convergence criterion, that checks are made to ensure convergence in the field of interest has also occurred. This appears to be particularly true for the determination of force and / or torque as these are particularly mesh dependent. Some of the checks, as recommended by Salon [124] include solving the same problem with several different meshes in order to
ensure convergence and, if possible, using different methods as discussed in section 3.4.3.3 (below).

3.4.3.2 Number of Time Steps

If analysis in the frequency domain is required, the number of samples must be at least equal to the Nyquist frequency to avoid aliasing and for this research the number of samples was, as a minimum, five times the Nyquist frequency [117, 121].

3.4.3.3 Calculation of Force and Torque

There are several methods of determining the force (and therefore torque) which results from a given flux density, such as Ampere’s force law and Maxwell’s stress method. However the most applicable to FEA is the Virtual Work Method as the starting point in FEA is often the minimisation of stored magnetic energy, which is less sensitive to local errors caused by poor meshing as it is a global quantity [124].

The virtual method calculates the force due to an MMF and involves performing two solutions, one a small distance from the actual position of interest and subtracting the two energies at these positions, then dividing by the small displacement. That is:

\[
F_s = \frac{W_2 - W_1}{\Delta S} \quad \text{Eq 10}
\]

Where

- \( F_s \) is the force in direction \( s \),
- \( W_1 \) is the magnetic co-energy at position 1
- \( W_2 \) is the magnetic co-energy at position 2
\( \Delta S \) is the small displacement in the direction of the force

However, the method as stated is somewhat problematic. The distance from the actual position of interest requires careful consideration as too small a distance results in loss of accuracy as does too large a distance. Also, the change in energy may not be entirely due to mechanical work as some could be from or to a source. However, these problems were overcome by Coulomb [125] who determined that a single solution would suffice if the energy function was directly differentiated with respect to a virtual distance. A full explanation can be found in [124].

In the Ansys Maxwell FEA software, however, rather than displace the actual object, only the tetrahedra that lie along the outside surface are virtually distorted to reduce computation time [126].
Chapter 4

Experimental Test Rig

Section 2.1.7.3 presented the documented options for experimentally measuring cogging torque and also discussed the difficulties associated with each of these.

This chapter will describe the design process for the experimental test rig used to measure and acquire the experimental cogging torque data.

4.1 Background

The need to accurately measure PMSM cogging torque within Charles Darwin University existed prior to the commencement of this research, and an experimental test rig had been previously developed jointly by the author and others [127-129]. However, analysis conducted by the author on the developed rig with the selected sensors proved that it was unable to isolate motor torque from other out of balance forces and as such was found unsuitable for conducting the required testing. This is discussed in further detail in Appendix C - Experimental Test Rig.
Analysis and subsequent modifications to this early version of the test rig were conducted by the author and these formed sections of the paper:


The following sections of this chapter are based on the published work.

### 4.2 Measuring Cogging Torque

This section presents the PMSM investigated and the requirements of the experimental test rig as well as the design process.

#### 4.2.1 Motor Description

The research motor being investigated was a 750 W, 3 Nm axial flux PMSM which employed fractional pitch and pole skew as the primary means of reducing cogging torque. It had 20 poles and 24 slots resulting in a lowest common multiple of 120 and the trapezoidal poles were skewed approximately 9°, or one full slot width. The motor was predominantly used as a high efficiency pool pump drive, however, its application was to be widened to include more noise sensitive situations.

Full specifications are in Appendix A – Test Motor Specifications.
4.2.2 Overall System Requirements

To accurately measure the cogging torque, the experimental apparatus must meet certain criteria. These include:

1. The measured cogging torque must not be influenced by resonant frequencies within the operating range.
2. Suitable sensor selection and mounting location that is not affected by forces other than the motor torque.
3. System resonant frequencies at least 5 times higher than the highest measured frequency to ensure measurement linearity [117] (p 166).
4. Light and stiff design of the rotating elements to maximise the fundamental resonant frequency and therefore maximise the potential sampling rate.
5. No significant influences on measured torque from bearings, friction elements or drive torque ripple.
6. If using an inline torque sensor, then minimum inertial loads to avoid the attenuation of any pulsating torque. (Does not apply if a stiff reaction torque sensor is used).

4.2.3 Mechanical Design

This section will detail the requirements for the mechanical aspects of the cogging torque test rig.

Accurate cogging torque measurement requires the experimental apparatus’s transfer function to be time and frequency independent over the operating range. Therefore, it is important that bearing loads, internal or external drive forces and resonant frequencies do not affect the measurements.
In general terms, there are two possible layout options, both of which have certain advantages and disadvantages over each other. These are

1. Single shaft / concentric bearing arrangement or
2. Twin shaft / separate bearing arrangement

Both of these options along with their benefits and drawbacks will now be presented.

### 4.2.3.1 Single Shaft Design.

The single shaft concentric bearing arrangement consists of the rotor bearings being mounted internally to the stator and an inline torque sensor between the rotor and a brake as depicted in Figure 4.1. One advantage of this configuration is that axial concentricity is built into the system and therefore no alignment is required.

![Figure 4.1. Single shaft experimental test rig design [130].](image)

The equation of motion for this design is

\[
T_{brake} + k_{sensor} = T_{motor} + b_{bear}
\]
Experimental Test Rig

\[ k_{sensor}(\theta_R - \theta_B) - b_{bear}\dot{\theta}_R + T_{motor} = J_R\ddot{\theta}_R \] \hspace{1cm} \text{Eq 11}

where:

- \( k_{sensor} \) = torque sensor stiffness
- \( \theta_R \) = rotor angular position
- \( \theta_B \) = brake angular position
- \( b_{bear} \) = bearing viscous friction coefficient
- \( T_{motor} \) = motor torque output
- \( J_R \) = rotor angular moment of inertia

Measured torque information is derived from the term with the sensor stiffness, therefore:

\[ T_{measured} = k_{sensor}(\theta_R - \theta_B) \] \hspace{1cm} \text{Eq 12}

\[ T_{measured} = J_R\ddot{\theta}_R + b_{bear}\dot{\theta}_R - T_{motor} \] \hspace{1cm} \text{Eq 13}

where:

- \( T_{measured} \) = measured torque

Equations 12 and 13 show the dependence of the measured torque not only on motor torque, but also the inertial and bearing torques. Furthermore, any external drive loads on the rotor would also contribute to an additional force/torque influencing the measured torque, leading to further reductions in measurement system independence. To avoid these potential problems, a new design was developed as depicted schematically in Figure 4.2.
4.2.3.2 Twin Shaft Design.

In this alternate layout, the stator and rotor are mounted on separate shafts and a reaction torque sensor is used. The equation of motion is now:

\[ k_{\text{sensor}}(\theta_S) - b_{\text{bear}}\dot{\theta}_S + T_{\text{motor}} = J_S\ddot{\theta}_S \]  

where:

\[ J_S = \text{stator angular moment of inertia} \]
\[ \theta_S = \text{stator angular position} \]

Figure 4.2. Twin shaft experimental test rig design [130].

The stator is connected to a stiff piezoelectric reaction sensor and is therefore, to all intents and purposes, not moving, consequently \( \theta_S \approx \dot{\theta}_S \approx \ddot{\theta}_S \) are all small enough to not affect the measurement system and can be considered equal to zero (\( \theta_S \approx \dot{\theta}_S \approx \ddot{\theta}_S \approx 0 \)). Thus:
Equations 15 and 16 show that the measured torque and motor torque are independent of any other parameters, therefore, a two shaft design, with the stator and rotor mounted separately, avoids the issues related with bearing friction, bearing fault frequencies and rotating inertia. Additionally, the implementation of an external indirect drive on the rotor will not influence the cogging torque measurement as any additional forces are reacted by the rotor bearings only.

One disadvantage of a two shaft configuration is the need to pay careful attention to alignment of the two shafts, as axial concentricity is no longer guaranteed. Conversely, this disadvantage is also an advantage for the investigation of angular misalignment and eccentricity as these ‘faults’ can easily induced. Single shaft design configurations are difficult to implement deliberate and known misalignments with the need for complex mechanical modifications such as offset bearing holders [103].

The advantages of the twin shaft design over a single shaft configuration were deemed to outweigh the potential alignment disadvantages and thus this was the selected design.

4.2.3.3 Repeatability

In section 4.2.3.2 above, the advantages of a two shaft design were presented. This section will discuss another important aspect of the experimental test rig mechanical design: repeatability.
The purpose of the experimental test rig was to measure cogging torque data for different rotor and stator combinations. For the results to be comparable, changing rotors or stators should have minimal impact on the data obtained. That is, the process of removing and installing a rotor or stator should not influence the measurement. This can be achieved by maintaining stator and rotor relativity between changing parts. The important dimensions to control are:

- Air gap between the stator and rotor
- Axial alignment between the stator shaft and rotor shaft
- Parallelism of stator face to rotor face

The nominal air gap for the test motor was 1 mm. As such, removal or replacement of either stators or rotors required initial separation of the two components. This was achieved by mounting the stator shaft assembly on a sliding platform. The platform was constrained by two rails that ensured the axial alignment and parallelism of the stator assembly to the rotor shaft while being able to extract the stator shaft to allow part changes. This was achieved by increasing the air gap by sliding the platform away from the rotor shaft. During reassembly, the air gap was set by a mechanical hard stop at the rotor side of the platform rails. These details are shown in Figure 4.3.
4.2.4 Sensor Selection

Chapter 2.2.1 presented the documented sensors suitable for measuring the cogging torque of PMSM, namely strain gauge or piezoelectric sensors. In order to maximise the bandwidth of the measurement system, the stiffest possible sensor coupled to a well-designed rig is required. Additionally, to avoid any mass attenuation of the cogging torque caused by inertial loads, a reaction torque sensor was to be used on the stator.

Piezoelectric sensors are usually far stiffer than strain gauge sensors as can be seen by the comparison between the torsional stiffness of two available reaction torque sensors. The Himmelstein RTM 2010(12-1) strain gauge sensor has a torsional stiffness of 11,128 Nm/rad [131] while the Kistler 9339A piezoelectric sensor has a torsional stiffness of 96,000 Nm/rad [132], or 8.6 times stiffer. As the fundamental frequency of the sensor is determined by the square root of the stiffness, the Kistler alternative will allow a measurement sampling rate of up to
\[ \sqrt{8.6} \] or nearly 3 times higher before breaching the requirement that the measuring sample rate be 5 times lower than the lowest fundamental frequency [117].

For these reasons, a Kistler 9339A piezoelectric sensor with a Kistler 1 channel 5073A111 charge amplifier was selected [132, 133].

### 4.2.5 Sensor Location

In addition to the selection of the most appropriate sensor, the positioning of the sensor on the test rig and its subsequent loading is critical to ensuring that the only forces that influence the sensor are those due to motor torque. This is achieved in two ways —

1. Selection of a sensor that is unaffected by cross talk forces, that is, torque measurement is insensitive to all non-torque forces.
2. Test rig design to minimise the magnitude of any cross talk forces at the torque sensor.

Item 1 was addressed in section 4.2.3 and this section will address the second of these two points.

Crosstalk between axial, shear and bending moments on the Kistler 9339A torque sensor are specified [132] with the following figures —

- Axial force \( (F_z) \) on torque \( (M_z) \): ±0.05 mNm/N (1 in 20,000)
- Shear force \( (F_{x,y}) \) on torque \( (M_z) \): <0.3 mNm/N (1 in 3333)
- Bending moment \( (M_{x,y}) \) on torque \( (M_z) \): <8 mNm/N (1 in 125)
The dynamic axial loading on the sensor is negligible as a high precision roller bearing is used to take the axial forces created by the attraction of the PMs to the stator. Any static axial loading which may remain from the installation is not measured as piezoelectric sensors are incapable of measuring average or DC forces (these reduce to zero over time).

The shear force and bending moment sensor crosstalk figures can be further improved by placing the sensor rearward of the high precision bearing thus providing the sensor with a favourable leverage ratio, an example of which is shown in the free body diagram imposed over a CAD model of the stator side of the experimental test rig (Figure 4.4).
Figure 4.4. Schematic of the stator section of the test rig with free body diagram imposed to show the resultant forces on the torque sensor as a result of an axial forces applied by one pole on the stator. The use of two poles assists in reducing the magnitude of shear and bending moment forces at the torque sensor.

As a result of the favourable leverage and bearing loads being applied to the sensor crosstalk data, the effective crosstalk for the sensor become

- Axial force is insensitive.
- Shear force at the stator face \((F_{X,Y})\) on torque at the sensor \((M_z)\): <0.143 mNm/N (1 in 7000 or 0.01%).
- Bending moment at the stator face \((M_{X,Y})\) on torque at the sensor \((M_z)\): <3.81 mNm/N (1 in 262 or 0.4%).

### 4.2.6 Drive Options

In section 2.2.2 the problems associated with various drive methods was reviewed. This section will describe the selected method.
The driving of the rotor to induce cogging torque can be achieved either by the motor itself or by an external drive. The significant disadvantage of using the motor itself (that is, internal drive configuration) is that the torque sensor will measure pulsating torque, consisting of both cogging torque and torque ripple. This option would require the cogging torque and torque ripple to be decoupled to determine only the cogging torque. By using an external drive, there are two advantages, one torque ripple is not present, therefore all the pulsating torque is cogging torque and therefore no decoupling is required. A significant further advantage relates to the fact that cogging torque is generated from the interaction of the PMs with the stator’s changing ferromagnetic reluctance. As copper windings do not alter the stators magnetic attributes, they do not affect cogging torque. Thus, unwound stators can be successfully tested for cogging torque, somewhat simplifying the testing and lowering the cost of required production parts.

Drive options for a twin shaft configuration test rig are somewhat less complicated than they may be for a single shaft design as any external loading induced by an external drive option on the rotor is totally independent of the stator. Thus, external drive loadings do not affect the torque measurements.

The external drive would only need to provide sufficient torque to free spin the PMSM. At these low torque levels the possible drive options could include rubber ‘O’ rings as well as other light duty drive specific belts. However, an advantage of an ‘O’ ring drive belt are numerous:

- Simple groove design on the test rig for the belt
- Cheap and readily available in a variety of lengths
- Elastic connection between the external drive and the experimental apparatus will not transfer any vibration
- Attenuation of any possible external drive sourced pulsating torque.
For these reasons, the selected drive option consisted of a separate DC motor connected to the rotor via a rubber ‘O’ ring drive belt.

4.2.7 Resonant Frequencies

Resonant frequencies within the sampling range can affect the torque measurements. To ensure measurement system linearity error is less than 5%, the sampling frequency should be 5 times lower than the lowest resonant frequency [117].

Thus, there are two outcomes required. Firstly, the experimental test rig needs to be designed to have the highest possible resonant frequency so that sampling rates can be maximised. The second is to measure the completed system resonant frequency to determine the reliable sampling frequency and set this as an experimental apparatus limitation.

The stator and rotor components are predetermined and cannot be altered; therefore their resonant frequencies will determine the maximum possible upper limit for which all other test rig components can be designed.

4.2.7.1 Torsional Resonant Frequency

The stator’s torsional resonant frequency was determined by measuring the inertia of the stator ($I_s$) using a trifilar [134] as shown in Figure 4.5 in conjunction with the stiffness of the selected torque sensor ($k_{sensor}$) [31]. As the dimensions of the trifilar rig are known, the inertia of the plate can calculated and then subtracted from the total inertia of the plate and stator, providing the inertia of only the stator.

Assuming a simplified single degree of freedom system, the fundamental torsional frequency is [135]:
The selected sensor had a specified stiffness of 96,000 Nm/rad and the measured inertia of the stator was $2.14 \times 10^{-3}$ kg.m$^2$ giving a fundamental torsional frequency of 1066Hz. Therefore the maximum sample rate for accurate measurement is five times less than this, as discussed in section 2.2.4, at 213Hz.

\[
\omega_n = \sqrt{\frac{k_{\text{sensor}}}{J_{\text{stator}}}} \quad \text{Eq 17}
\]

4.2.7.2 Modal Analysis

The test rig was designed using 3D CAD and the resonant frequencies of all components analysed using FEA to ensure they were as high as practicable to maximise sampling rates. The critical area of the design was the torsional resonance of the stator and stator shaft assembly, for which the FEA model is shown in Figure 4.6. The selected bearing to be installed on the rig was

Figure 4.5. Trifilar rig for stator inertia measurement [130].
incapable of providing moment support thus the frictionless support B required careful design to accurately reflect this. The model was developed with “structural steel” as the selected material with the exception of the stator and torque sensor. The torque sensor modulus of elasticity was modified to match the stiffness specified on the data sheet and the stator density was reduced so that the moment of inertia matched the experimentally determined value (section 4.2.7.1). The analysis predicted a torsional resonant frequency of 498Hz.

Figure 4.6. ANSYS model of the stator assembly showing the bearing support (B) and torque sensor support (A).

4.2.7.3 Analytical Results

To validate the FEA results, an analytical solution was calculated. The stiffness of multiple elements is the inverse sum of all the components such that:
\[
\frac{1}{K_{total}} = \frac{1}{K_{sensor}} + \frac{1}{K_{shaft}} \quad \text{Eq 18}
\]

Where:

- \(K_{total}\) is the stiffness of the total system
- \(K_{sensor}\) is the sensor stiffness
- \(K_{shaft}\) is the shaft stiffness

The stiffness of the sensor is specified at 96,000 Nm/rad, while the shaft stiffness was calculated from:

\[
K_{shaft} = \frac{GJ}{L} \quad \text{Eq 19}
\]

Where

\[
J = \frac{\pi d^4}{32} \quad \text{Eq 20}
\]

- \(G\) = the modulus of torsional rigidity, which for structural steel is 80 GPa.
- \(d\) = the diameter of a solid shaft.
- \(L\) = shaft length
- \(J\) = angular moment of inertia

Thus the total stiffness of the stator sub assembly was calculated to be 38,261 Nm/rad.

The moment of inertia of multiple components is the sum of the individual elements. The moment of inertia for the stator was measured as \(2.14 \times 10^{-3}\) kg.m\(^2\) (from section 4.2.7.1) while the stator shaft was determined to be \(1.89 \times 10^{-3}\) kg.m\(^2\) from 3D CAD.
Thus the analytically determined torsional resonant frequency was calculated as 490Hz, which is less than 2% different from the FEA predicted value.

### 4.2.7.4 Resonant Frequency Summary

The lowest resonant frequency of the experimental test rig was found to be the torsion of the stator shaft at 490 Hz. This was determined and validated through FEA and analytical analysis. Heins et. al. [130] also conducted an experimental analysis on the same components which measured the lowest resonant frequency at 518Hz. Comparing the available data in Table 2 shows the three methods are within 4.3% of each other.

<table>
<thead>
<tr>
<th>Method</th>
<th>Result</th>
<th>Difference</th>
</tr>
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<tr>
<td>Experimental</td>
<td>518 Hz</td>
<td>-</td>
</tr>
<tr>
<td>Modal analysis</td>
<td>498 Hz</td>
<td>2.7%</td>
</tr>
<tr>
<td>Analytical calculations</td>
<td>490 Hz</td>
<td>4.3%</td>
</tr>
</tbody>
</table>

Conservatively using the lowest of these values and given that the area of interest with the research motor is up to 120 orders, the permissible sampling frequency could be \((490/5)/120 = 0.80\) Hz or approximately 50 RPM. Therefore, torque measurements at speeds below 50 RPM will not be influenced by resonant frequencies and harmonic analysis up to 120 orders should be reliable. As cogging torque measurements require the PMSM to be unpowered and driven from an external source, it is of no concern that 50 rpm is well outside the normal operating speed range of the motor as the cogging torque itself is independent of rotational speed. The other advantage of a slow rotation speed is that eddy current losses have negligible impact on the test measurements.
4.3 Data Capture

A data acquisition system was required to concurrently capture data from the encoder and torque transducer so that the torque at specific rotor angles could be determined.

The following sections will present the hardware and software implemented on the experimental setup.

4.3.1 Data Acquisition Hardware and Software

Existing hardware and software licences for National Instruments products permitted rapid implementation, therefore a National Instruments PCI6259 D/A data acquisition card was used coupled to a PC running Labview™ using a series of virtual instruments. Data was acquired in a batch which was then saved to a text file. Twenty complete revolutions were recorded and averaged per test.

Encoder

Existing hardware allowed the utilisation of a 12 bit (4096 count) hollow shaft encoder, BEI Model HS35 Absolute Encoder that fitted over the rotor shaft to provide approximately 204 counts per pole / slot interaction.

Torque Sensor

In section 2.2.1 the options for torque sensors were reviewed with the evidence suggesting a piezoelectric reaction torque sensor was required. Based on the comprehensive and detailed specification sheet [132] which stated sensor sensitivity to crosstalk forces, a Kistler 9339A piezoelectric reaction torque sensor capable of measuring ±10 Nm was interfaced with a Kistler charge amp type 5073 [133]. This setup is capable of accurately measuring <±0.18 Nm while
still being able to withstand 5 kN of axial load, sufficient to overcome the rotor to stator attractive force should the sensor be subject to this. Also, as the test motor has a maximum torque output of 3 Nm, the next more sensitive sensor (±1 Nm) would have been overloaded under power.
Data Analysis

Matlab™ was used to process and analyse the captured data.

4.4 Additional Requirements for Rotor / Stator Misalignment Testing

Generally the requirements for a PMSM test rig would be to ensure as near to ideal alignment of the rotor and stator, however, to investigate misalignment known offsets are required. By utilising a two shaft experimental test rig design, as well as obtaining the benefits as outlined in section 4.2.3, it was relatively straightforward to acquire data for non-aligned situations. Specifically:

1. Static eccentricity
2. Dynamic eccentricity
3. Static angular misalignment
4. Dynamic angular misalignment

This section will detail the modifications made to the test rig to accurately acquire the experimental misalignment data.

4.4.1 Static Eccentricity

The experimental test rig was modified to allow the rotor section to be moved in the horizontal plane inducing rotor / stator static eccentricity misalignment (Figure 4.7). Jacking bolts were added to the sides of the cast iron base to precisely control the horizontal movement of the aluminium supports to which the pillow block bearings were bolted. Spacers were added to the front bearing to ensure a consistent rotor / stator air gap was maintained. Both bearings were indexed fully to one side ensuring there was significant offset of the rotor to the
stator. The rotor was then minor adjusted to eliminate any angular misalignment. After each test, both jacking bolts were indexed a quarter of a turn or 0.125 mm until the bearings were at the other side of the base block, thereby ensuring that the aligned position was passed within 0.0625 mm (half of 0.125 mm). It should be noted that the same effect could have been achieved by offsetting the stator, however, on this test rig, the rotor was easier to adjust and therefore the preferred option.

Further details of the procedure for obtaining the misalignment data is presented in chapters 5.4 to 5.7.

Figure 4.7. Experimental test rig showing jacking bolts, pillow block bearing anchor bolts and spacer plates.
4.4.2 Dynamic Eccentricity

Dynamic eccentricity was induced by machining a slightly larger rotor backing plate (110 mm diameter) and laser cutting acrylic pole position formers with an offset centre mounting hole (Figure 4.8). Each of the formers had the centre hole offset by an incremental 0.1 mm, therefore dynamic eccentricity from 0 mm (aligned) to 2.2 mm of offset was measureable.

Figure 4.8 Dynamic eccentricity induced by using a larger rotor backing plate and laser cut magnet holders. This is former 20 which has 2.0 mm of offset.

4.4.3 Static Angular Misalignment

Static angular misalignment was induced by adding shims behind the stator. Stainless steel shims from 0.05 mm to 0.75 mm in 0.05 mm increments were placed between the stator and the stator backing plate to angle the stator to the rotor (Figure 4.9). By knowing the position of the shim and the geometry of the stator the induced angle can be calculated.
4.4.4 Dynamic Angular Misalignment

Similar to inducing static angular misalignment, dynamic angular misalignment was generated by inserting shims between the rotor backing plate and the rotor shaft as can be seen in Figure 4.10. For this, the shims were cut so that only 5 – 10 mm was inserted.
4.5 Summary - Experimental Test Rig

The final design of the experimental test rig consisted of production PMSM rotors and stators and:

- A twin shaft design with rotor and stator shafts mounted separately. Production rotors and stators (the motor) were bolted to rotor and stator backing plates.
- A rotor shaft externally driven by a DC motor and rubber ‘O’ ring.
- Piezoelectric torque sensor positioned to reduce cross talk.
- A large cast iron mounting base to mass attenuate vibrations.
- Jacking bolts were added so that the four different misalignments could be induced.
Figure 4.11 and Figure 4.12 below show the final design of the experimental test rig.

Figure 4.11. Stator assembly with the key components labelled.

Figure 4.12. Plan view of the experimental setup with the single slot stator installed. Encoder and external drive as labelled.
Chapter 5

Implemented Methods for Decoupling Contributions to Cogging Torque

Chapter 3 presented previous work on the methods available for predicting the cogging torque for PMSMs. While the methods discussed were suitable for predicting the cogging torque output for PMSMs with perfect construction, they were generally not capable of predicting cogging torque for situations where manufacturing induced variations existed (section 3.3). To overcome these issues, this chapter presents the new methods developed that predict the major contributors to unexpected cogging torque. These methods allow statistical analysis of the effect of a range of manufacturing errors rather than a single solution as is possible with FEA.

Section 2.1.7.1 identified that stator affected harmonics are multiples of the number of poles and rotor affected harmonics are multiples of the number of slots. Figure 5.1 is an example of an experimentally derived cogging torque
waveform while Figure 5.2 is the same data in the frequency domain which shows the rotor and stator affected harmonics are the most significant. For these reasons, the analysis starts with the rotor and stator decoupling method in section 5.1. Subsequently the method used for further decoupling the rotor causes into pole placement and pole strength contributions is presented in section 5.2. Finally the methods used to analyse the effect of static and dynamic angular misalignment are presented in section 5.4 and 5.5 respectively and rotor to stator eccentricity misalignment are discussed in sections 5.6 (for the static situation) and 5.7 (for the dynamic situation).

![Sample Cogging Torque - Time Domain](image1)

![Sample Cogging Torque - Time Domain Expanded View](image2)

Figure 5.1 Sample cogging torque waveform from a random rotor / stator combination, measured experimentally.
Figure 5.2 Typical example of the order analysis for a PMSM with 24 slot, 20 pole stator/rotor combination showing the individual harmonics affected by inaccuracies in the stator and rotor and the native component, which is always present, even in perfectly manufactured PMSMs. Top pane shows orders from 0 to 120, lower pane shows the ‘critical’ orders from 18 to 26 which also clearly indicates the sidebands of the stator and rotor affected harmonics.
5 Implemented Methods for Decoupling Contributions to Cogging Torque

5.1 Decoupling Stator and Rotor Contributions

In section 2.1.7.1 it was identified that stator affected harmonics are multiples of the number of poles and rotor affected harmonics are multiples of the number of slots. Based on this insight, the developed method calculates the RMS cogging from the rotor and stator affected harmonics and compares it to the total RMS cogging. This allows a percentage contribution from each to be calculated. What is not due to the stator or rotor will temporarily be categorised as ‘other’.

An order analysis of the cogging torque for each rotor and stator combination was created and the magnitude of the harmonics correlating with the number of poles and slots determined, for the motor studied the 20th (stator affected harmonic) and 24th (rotor affected harmonic). The magnitude of the relative harmonic determines the fraction to which the rotor and stator contribute to the RMS cogging torque (Figure 5.1 and Figure 5.2). This can be expressed as a complex Fourier series [61]:

\[
\hat{T}_\theta = \sum_{n=-\infty}^{\infty} \hat{T}_n e^{jn\theta}
\]

Eq 21

where:

\( \hat{T}_\theta \) = the normalized cogging torque at angle \( \theta \)

\( \theta \) = rotor mechanical position [rad]

\( n \) = harmonic number

\( \hat{T}_n \) = normalized cogging torque Fourier series coefficient

\( \hat{T}_n = \frac{T}{T_{\text{rated}}} \), where \( T_{\text{rated}} \) is rated motor torque which in the production motors used here is 3 Nm.
All measured cogging torques will have the mean subtracted to correct for any sensor drift over time and to eliminate the combined DC effects of iron losses and windage.

Further:

\[
\hat{T}_n = \frac{1}{2\pi} \int_{\theta=0}^{2\pi} T_\theta e^{-jn\theta}
\]

Eq 22

Rotor affected harmonics occur at a multiple of the number of slots \((N_s)\), such that:

\[
\hat{T}_{\text{rms, rotor}} = \sqrt{2 \sum_{n=N_s,2N_s, \ldots \text{excluding multiples of LCM}} |\hat{T}_n|^2}
\]

Eq 23

Likewise, stator affected harmonics occur at a multiple of the number of poles \((N_p)\), such that:

\[
\hat{T}_{\text{rms, stator}} = \sqrt{2 \sum_{n=N_p,2N_p, \ldots \text{excluding multiples of LCM}} |\hat{T}_n|^2}
\]

Eq 24

The native harmonics occur at multiples of the LCM such that:

\[
\hat{T}_{\text{rms, native}} = \sqrt{2 \sum_{n=LCM,2LCM, \ldots} |\hat{T}_n|^2}
\]

Eq 25

All remaining harmonics can be expressed as:

\[
\hat{T}_{\text{rms, stator}} = \sqrt{2 \sum_{n=1,2, \ldots \text{excluding multiples of } N_s \& N_p} |\hat{T}_n|^2}
\]

Eq 26
5.1.1 Experimental Design

In order to validate the independence of stator and rotor affected harmonics, all possible combinations of 10 production stators and 10 production rotors had their cogging torque measured. These time domain waveforms were then analysed in Matlab™ to produce cogging torque waveforms in the frequency domain for all of the 100 combinations of 10 stators and 10 rotors. The magnitudes of the harmonics corresponding with the number of poles and slots were measured and these plotted to visualise the relationship between parameters.

5.1.1.1 Traceability

The testing of known rotor and stator combinations was imperative for assessing the relationship between stator affected and rotor affected harmonics. To ensure the parts were tested in the correct order they were numbered with an engraver and all the tests were conducted on the same day to minimise measurement error. Each test consisted of an average of 20 revolutions of recorded data to provide some degree of redundancy should erroneous data be subsequently detected, however, this proved unnecessary as later analysis showed undetectable variations between each revolution of data capture. However, the additional 23 seconds required to acquire the additional 19 revolutions was deemed a reasonable investment at the time.

The data files were loaded into Matlab for analysis. Each file consisted of 4096 encoder points with the cogging torque measured at each point.
5.2 Decoupling Rotor Pole Strength and Placement

The previous section separated cogging torque harmonics due to the rotor from those due to the stator and chapter 2.1 highlighted the importance of accurate pole placement and uniform pole strength. This section presents a method to further classify the rotor affected harmonics into those caused by pole misplacement and those caused by non-uniform pole strength.

5.2.1 Method Overview

The flow chart in Figure 5.3 provides an overview of the method for decoupling pole placement and strength.
5 Implemented Methods for Decoupling Contributions to Cogging Torque

Experimentally derived production rotor cogging torque from a single stator slot (section 5.2.3)

Determine the magnet positions from the zero torque point (section 5.2.3)

Use superposition to create cogging torque waveforms identical to the production rotors, but with perfect magnet strength (section 5.2.4)

Perform a least squares minimisation with the production and superposition data to determine the magnet strength variation. (section 5.2.5)

Magnet position histogram. (Figure 6.23)

Magnet relative strength histogram. (Figure 6.24)

Figure 5.3 Flow chart for the method developed to decouple pole strength and placement.
5.2.2 Assumptions

This method assumes

- Standard superposition assumptions as discussed in section 3.1
- Magnet strength variation results only in alterations to the scale of the cogging torque waveform
- Pole skew variation has little or no effect on cogging torque waveform
- Pole saturation is not critical for the rotors tested
- A pole transition, that is, two poles passing a single stator slot can be used as the foundation waveform to create complete motor cogging torque waveforms.

Data to validate these assumptions are presented in section 6.2.1.

5.2.3 Determine Pole Placement

Sections 2.1.1 and 2.1.7 discussed the importance of accurate pole placement and uniform pole strength for effective reduction of cogging torque in fractional pitch PMSMs. This section presents the method used for decoupling pole strength and placement.

It may seem beneficial to be able to test any stator and rotor combination to determine the pole strengths and locations, however, in this case this is not feasible. To fully define the magnet placement and relative magnet strengths for the research motor \( N_p = 20 \), 38 parameters of independent rotor information are required, that is 19 parameters for the pole relative positions and 19 for the pole relative strengths. As there are 24 slots in the stator and as rotor data is contained in harmonics of the number of stator slots, a total of 912 \((24 \times 38)\) total harmonics are required to resolve sufficient information for the desired decoupling.
Figure 5.4 shows a typical order analysis for a rotor and stator combination. The lack of information available beyond the 200\textsuperscript{th} order is clearly visible. When looking at the log scale in the lower pane, the magnitude of harmonics above the 400\textsuperscript{th} is approximately one one hundredth to one ten thousandth of that below 150 orders, and this is the reason decoupling pole strength and pole placement from a multi-slot rotor is unfeasible.
5 Implemented Methods for Decoupling Contributions to Cogging Torque

Figure 5.4. A typical order analysis for a rotor / stator combination (top pane) with the same plot shown on a log scale (lower pane).

Zhu et. al. [7] utilised a single stator slot in FEA to synthesise an analytical cogging torque waveform and this lead to an alternate approach which is to experimentally test each rotor against a single slot stator (Figure 5.6), thereby eliminating the influence of the variations in individual stator slots. The pole
relative positions can be determined from the cogging torque waveform by analysing the location of the zero torque points from the rotor revolving passed a single slot stator.

There are two zero torque points associated with each pole pair. The first occurs when the pole is directly over a slot and is created due to the pole being equally attracted to the stator material on either side of the slot as graphically represented by pole 2 in Figure 5.6. At this position, the zero torque is due to only one pole and neighbouring poles have limited influence on the position. The second zero torque point occurs when the gap between two poles is approximately over the centre of the stator slot and is due to the equal attraction of the two neighbouring poles to the stator material either side of the stator slot. This is graphically represented by the gap between poles 1 and 2 in Figure 5.7. As this zero torque point relies on the equal but opposite attraction between two neighbouring poles, its position is also dependent on the relative strengths of these poles and therefore shifts depending on pole strength variation.
5 Implemented Methods for Decoupling Contributions to Cogging Torque

Figure 5.6 Graphical representation of poles passing over a single stator slot at the pole over slot zero torque point, the exact position of which is comparatively invariant on the relative strength of poles 1, 2 or 3. This allows the position of pole 2 to be accurately determined.

Figure 5.7 Graphical representation of poles passing over a single stator slot at the gap over slot zero torque point, the exact position of which depends on the relative strengths of poles 1 and 2.
The 10 production rotors were tested against the single slot stator and the cogging torque measured. The pole over slot zero torque points were determined and subsequently the location and spacing between poles determined. These data can then be used in conjunction with the analytical method discussed in the next chapter, to determine the relative magnet strengths.

### 5.2.4 Uniform Pole StrengthCogging Torque Waveform from Superposition

This section determines the pole strength variation by analytically creating a cogging torque waveform with poles in an identical position to the production rotors, but with poles of identical strength. The simulation method utilised superposition of a library of pole transition over slot waveforms selected and assembled in the correct order to predict the complete waveform.

In order to build the library of waveforms, all combinations of pole spacing variations were simulated using 3D FEA.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Determine the required range of magnet spacings to simulate the PMSM in question.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In this case this was:</td>
</tr>
<tr>
<td></td>
<td>Magnet spacing from 16° to 20° in 1° increments. 1° increments provided sufficient</td>
</tr>
<tr>
<td></td>
<td>data for accurate cubic spline interpolation (detailed in step 3) while not</td>
</tr>
<tr>
<td></td>
<td>significantly extending the FEA computation time.</td>
</tr>
</tbody>
</table>
### Step 2
**In Ansys Maxwell**

Obtain the 3D FEA data.

In this case, to ensure a high degree of accuracy, the data was obtained magnetostatically using adaptive mesh control to an accuracy of 0.4% energy error.

### Step 3
**In Matlab**

Cubic spline interpolate the FEA data over a finer mesh to allow better representation of the actual motor to be simulated.

In this case magnet spacing was refined from 1° increments to 0.05° increments.

Figure 5.7 shows the data direct from FEA and also after having been cubic spline interpolated.
Figure 5.8. FEA data (top pane) cubic spline interpolated from 1° pole spacing to 0.05° and trimmed (lower pane).

Step 4
In Matlab
Create a multi-dimensional matrix that incorporates all possible combinations of cogging torque data. This becomes the library of waveforms.
### 5 Implemented Methods for Decoupling Contributions to Cogging Torque

**Step 5**  
In Matlab

Use the library of waveforms as a lookup table to generate complete PMSM cogging torque waveforms.

See example in figure 5.8, pane 1.
5.2.5 Decoupling Pole Strength and Placement

The pole locations were determined by detecting the pole over slot zero torque point as outlined by the method presented in section 5.2.3 and these data were
then used to select the same width of pole transition over single slot waveforms from the FEA library. Each of the selected pole transitions was then positioned to the same location as the production rotors’ twenty poles, creating a matrix of superposition data consisting of one waveform for each pole pair, however, the superposition created waveform has uniform pole strength.

The pseudo inverse of these matrices is multiplied by the measured torque, providing a least squares minimization vector of the pole strengths.

\[
\begin{align*}
X\tilde{y} &= \bar{t} \\
X^+X\tilde{y} &= X^+\bar{t} \\
\tilde{y} &= X^+\bar{t}
\end{align*}
\]

where:

- \( X \) = matrix of pole transition waveforms, each phased to a known position.
- \( X^+ \) = the Moore-Penrose pseudo inverse [136, 137] of \( X \)
- \( \tilde{y} \) = least squares minimization vector of strengths
- \( \bar{t} \) = torque vector
- \( \tilde{y} \) is now the best linear combination of waveform magnitudes, or pole strengths, that explains the actual production rotor’s cogging torque.
- \( \tilde{y}_{average} \) can then be determined and this is the average strength of all the 20 production poles.

With these strength vectors, there are four combinations that can be analysed:

1. production rotor cogging torque
2. cogging torque with pole strength variation removed
3. cogging torque with pole placement error removed
4. cogging torque with both pole placement error and pole strength variation removed

To determine each of the four combinations listed above, the following process is applied.

1. The product of $X \times \bar{y}$ will provide data on actual pole placement with non-uniform strength, thus showing the cogging torque with both pole placement error and pole strength variation. This represents the production rotors’ measured cogging torque.

2. The product of $X \times \bar{y}_{\text{average}}$ will provide data on actual placement with uniform pole strength, thus showing the cogging torque with only placement variation.

3. The product of $X_{\text{perfect}} \times \bar{y}$, where $X_{\text{perfect}}$ is the perfect placement of the poles (for the research motor investigated, 20 poles perfectly placed every 18°), will provide data on perfect placement with non-uniform pole strength, thus showing the cogging torque with only pole strength variation.

4. The product of $X_{\text{perfect}} \times \bar{y}_{\text{average}}$ will provide data on perfect placement with uniform strength, thus showing the best possible (i.e. minimum) cogging torque without any pole placement error and no pole strength variation.

An additional advantage of the above process is that strength and placement data is obtainable from one test without the need for expensive and sensitive Gauss meters.
5.3 Decoupling Misalignment Assembly Errors Overview

This section will introduce the simulation methods used to generate the numerical results and the experimental procedures for capturing the experimental data. The results from the testing are presented in chapter 6.

There are two possible angular misalignments in an axial flux PMSM and in this research they are referred to as static and dynamic angular misalignment as defined in section 2.1.7.3 and 2.1.7.4 respectively. Additionally there are two possible eccentricity misalignments, static and dynamic, which were described in section 2.1.7.5 and 2.1.7.6.

The following four sections, one for each of the assembly variations considered, will detail the hybrid FEA / analytical methods developed to assess the effects of these variations on cogging torque. It will list their assumptions as well as provide details of the FEA model used to obtain the base cogging torque waveforms. Finally, the experimental procedure will be explained with the results from the testing presented in chapter 6.

Work from these sections contributed to the publication:


5.4 Static Angular Misalignment

5.4.1 Simulation Method

The simulation method implemented was based on the approach previously presented in section 5.2 with one additional element. The previously discussed
library of waveforms (section 5.2.4) used for decoupling pole strength and placement consisted of a range of pole spacings to account for variations in pole transition widths. To simulate angular misalignment, air gap variation is also required. Therefore, the library was expanded from a 2D to a 3D lookup table to include air gaps in the range of 0.1 mm to 2.5 mm, which, for the PMSM under investigation, was sufficient to model ±0.8° of angular misalignment (the maximum practical before poles contact with the stator). The 3D FEA data was obtained in increments of 0.6 mm of air gap and this was cubic spline interpolated to 0.025 mm increments in Matlab.
Figure 5.10. Air gap variation was simulated in FEA at 5 increments of 0.6 mm (top), which was then refined by cubic spline interpolation and trimmed in Matlab to increments of 0.025 mm over the same range (0.1 mm to 2.5 mm) (lower).

To simulate static angular misalignment, an analytical expression for the air gap must be modelled which will be dependent on the motor geometry and the location of the axis of rotation of the misaligned components. Once the air gap
for each of the pole / slot interactions is known, an appropriate cogging torque waveform can be selected from the library and positioned in the known pole and slot locations.

![Diagram of air gap variation due to static angular misalignment](image)

**Figure 5.11** Schematic representation of air gap variation due to static angular misalignment in the axial flux PMSM investigated.

To determine the air gap for any pole / slot interaction, with reference to Figure 5.11, the following analytical equation was developed:

\[
Ag_s = \left( \frac{ID + OD}{4} \right) \times \sin(\alpha_s) \times \cos(\theta_s) + Ag_0
\]

Eq 28

Where:
\( A_{g0} \) = Nominal air gap, the design air gap assuming no misalignment which for the motor investigated was 1 mm.

\( A_{gs} \) = static air gap and is the sum of \( A_{g0} \) + Air gap variation (which may be positive or negative, depending on the angle)

\( \alpha_s \) = static angle to be simulated, for this research this was 0 to 0.8°

\( \theta_s \) = the angle of the position of the slot to be simulated. For the motor investigated, the ideal location of the slots was every 15°.

\( ID \) and \( OD \) are the internal and external diameters of the rotor and stator respectively.

Slot and pole positions are measured relative to a datum of a known slot and pole (position 0). Once the locations of all poles and slots was configured, the air gap for each pole at every slot location was calculated using Eq 28 and the waveform representing the calculated air gap was selected from the library of waveforms and positioned to the correct location, thus a matrix consisting of 20 x 24 waveforms was created, one for each pole / slot interaction. Figure 5.12 is the cogging torque for poles passing 4 individual slots positioned at the datum point, 90°, 180° and 285° and shows how air gap variation caused by static misalignment affects cogging torque. Note that if the 90° and 270° slots were presented the cogging would have been identical, thus to improve clarity the cogging from the slot at 285° was selected.
Figure 5.12. Simulation data for 20 poles / 1 slot for static eccentricity shows that each slot has a unique but constant air gap while the air gap varies for each pole. Note that the 270° slot interaction is not shown as it is identical to the 90° interaction. Rather, the slot interaction at 285° is shown so that it can be differentiated.

To simulate the effect of rotor and stator inaccuracy on static angular misalignment, 10,000 analytical cogging torque waveforms consisting of random rotors and stators (normally distributed) each with 5 dynamic angular misalignments (0° to 0.8° in 0.2° increments) were simulated (50,000 cogging torque waveforms in total). The standard deviations for the pole and slot placement distributions were selected so that they matched those of the production rotors and stators. Stator slot placement was selected based on the analysis of production rotors tested with a single magnet to determine the variation in slot placement.
Fast Fourier Transforms were performed on all waveforms and the effect of the misalignment on the affected orders plotted, with the results presented in chapter 6.3.

### 5.4.2 Assumptions

The simulation method described above has the following additional assumptions to those listed in section 5.2.1:

- Angular misalignment causes variations in the effective air gap of the pole / slot interactions. These angled interactions will be modelled with parallel pole / slot interactions with an equivalent mean radius air gap. This assumption will be quantified and validated in section 6.3.1 of the results chapter.
- Precise analytical air gaps are rounded to the nearest 0.025 mm to allow selection of an appropriate waveform from the cogging torque waveform library.

For this analysis, the axis of rotation of the rotor or stator to induce the angular misalignment was the Y axis of a coordinate system placed on the same plane as the poles and shown in Figure 5.13.
Figure 5.13 Axis of rotation of the rotor or stator was about the Y axis of a coordinate system placed parallel with the pole faces. Angular misalignment exaggerated for clarity.

5.4.3 Electromagnetic FEA Model

Chapter 3.4 reviewed others work and provided background information relating to electromagnetic FEA. For the reasons outlined at the conclusion of that chapter, magnetostatic FEA with adaptive meshing was preferred over time stepping FEA. To obtain the pole transition data in a minimum of time, a segment of a stator with master / slave boundaries was modelled (Figure 5.14). The mesh accuracy was set to a maximum limit of 0.4% energy error and analysis was conducted every 0.5° for 25 degrees (sufficient to include the zero torque points of both poles with the widest pole spacing, 20°).
Figure 5.14 Pole transition over slot FEA model. Note that this is a 90° segment with master / slave boundaries (not shown) to ensure flux return paths are accurate.
Figure 5.15 Pole transition over slot FEA model shown with master boundary. The slave boundary (not shown) is at the other end of the segment.

The FEA pole transition over single slot model conducted sweeps for pole widths of 16° to 20° in one degree increments and air gaps from 0.1 mm to 2.5 mm in 0.6 mm increments. This provided 5 sweeps per variable which was then cubic spline interpolated in Matlab™ to a finer mesh for improved accuracy (Figure 5.10).

The splined library of waveforms contained all possible combinations of air gaps and pole spacings allowing the assembly of PMSMs with any combination of pole placements and angular misalignments (both dynamic and static) within the range of the original data captured.

To validate the hybrid FEA / analytical method developed, a complete PMSM with known pole and slot placements was modelled in 3D FEA (Figure 5.16) and the cogging torque compared to that predicted by the analytical method. These results are presented in section 6.3.
5.4.4 Experimental Procedure

The experimental test rig was used to obtain experimental data to validate the analytical method and the FEA data. Initially, the rotor and stator were placed in the aligned position, both horizontally and angularly by using a combination of feeler gauges and dial gauges. To induce a known static angular misalignment, feeler gauges were inserted behind one side of the stator, as discussed in section 4.4.3. From the thickness of the gauges, an angular misalignment can be calculated. The cogging torque was measured for each of the thicknesses of gauge inserted behind the stator.

5.5 Dynamic Angular Misalignment

This section presents the analytical method, FEA model and experimental procedure used to acquire data. The results of the analysis are presented in section 6.4.
The static angular misalignment assumptions previously detailed in section 5.4.2 also apply to the dynamic situation.

5.5.1 Simulation Method

Dynamic angular misalignment was simulated in a manner very similar to that presented in chapter 5.4.1 for static angular misalignment, with the key difference relating to the analytical determination of the air gap. Dynamic angular misalignment occurs when either a rotor shaft is bent or the rotor bore is not perpendicular to the face of the rotor, both of which result in each pole having a constant but unique air gap (the rotating reference frame) and the slots seeing a sinusoidal variation in air gap as the rotor rotates.

The same library of waveforms created for the analysis of static angular misalignment can be utilised, only the calculation of the variation of air gap reconsidered. As such, the previously list assumptions (section 5.4.2) also apply.

To analytically determine the air gap, the following equation was developed (refer to Figure 5.17 below):

\[
Ag_d = \left( \frac{ID + OD}{4} \right) \times \sin(\alpha_d) \times \cos(\theta_p) + Ag_0
\]

Eq 29

Where:

\( Ag_d \) = the sum of \( Ag_0 \) + air gap variation (the variation may be positive or negative, depending on the angle)

\( \alpha_d \) = the dynamic angle to be simulated, for this research this was 0 to 0.8°

\( \theta_p \) = the position of the pole to be simulated. For the motor investigated, the ideal location of the poles was every 18°.
\[ A_{g_0} = \text{the design air gap assuming no misalignment, which for the motor investigated was 1 mm.} \]

ID and OD are the internal and external diameters of the rotor or stator respectively.

As dynamic angular misalignment is based on unique air gaps for each of the poles, it is independent of the slot location aside from the phasing required to correctly position the waveform for each of the slots. Figure 5.18 shows 20 poles passing a single stator slot positioned at 0, 90, 180 and 270°. The only difference between the slots is the phase.

Figure 5.17 Schematic representation of air gap variation due to dynamic angular misalignment in the axial flux PMSM investigated.
Dynamic Eccentricity - 20 Magnets Passed 1 Slot at 90° Intervals

Figure 5.18. Simulation data for 20 poles passing individual slots at 0, 90, 180 and 270°. As this represents dynamic angular misalignment, each of the slots has the same profile with an incremental 90° phase change. Air gap varies with slot number while each pole has a constant but individual air gap.
5 Implemented Methods for Decoupling Contributions to Cogging Torque

As was the case for static angular misalignment, 10,000 cogging torque waveforms consisting of random rotors and stators with 5 dynamic angular misalignments each (0° to 0.8° in 0.2° increments) were simulated (50,000 cogging torques waveforms in total). The data were fast Fourier transformed to the frequency domain.

The effect of increasing dynamic angular misalignment with rotor and stator variation on the affected orders in the frequency domain is shown in section 6.4.

5.5.2 Electromagnetic FEA Model

The same library of waveforms created for static angular misalignment presented in chapter 5.4.3 was used for dynamic angular misalignment, thus there is no specific dynamic FEA model required for the generation of the lookup table data.

A complete PMSM was modelled with 0.6° of dynamic angular misalignment and known pole and slot locations for validate the hybrid FEA / analytical method.

5.5.3 Experimental Procedure

The same procedure as outlined in section 5.4.4 above was followed with the exception that the feeler gauges were inserted behind the rotor rather than the stator as presented in chapter 4.4.4.
5 Implemented Methods for Decoupling Contributions to Cogging Torque

5.6 Static Eccentricity

5.6.1 Simulation Method

The simulation of static eccentricity utilised the same library of waveforms generated and used for both static and dynamic angular misalignment.

As the eccentricity varies with mechanical rotation of the rotor, an analytical expression for the degree of eccentricity was developed.

\[ Ecc(\alpha) = max(Ecc) \times \cos(\theta) \]  

Eq 30

Where:

- \( \theta \) = the mechanical angle of rotation
- \( Ecc(\alpha) \) = the eccentricity at any angle \( \alpha \)
- \( max(Ecc) \) = the maximum eccentricity to be analysed

Eccentricity changes the timing of certain pole and slot interactions. Consider four poles positioned every 90° mechanical around the stator with static eccentricity as shown in Figure 5.19. Poles 0 (0° reference position) and 2 are aligned with the x axis and have the maximum eccentricity (inner radius overhung on the stator ID and outer radius overhung on the stator OD respectively). Poles 1 and 3 are aligned but the timings of their interactions with the slots at the 90° and 270° locations are altered due to the shifted position of the stator – assuming counter clockwise rotation, pole 1s interaction will be advanced and pole 3s interaction will be delayed. Clearly shifting the stator to the left or rotating clockwise will reverse these timings.
Therefore, to accurately model static eccentricity, the analytical expression must account for both the timing change of the pole / slot interactions and changing eccentricity.

![Diagram of four poles equally spaced](image)

**Figure 5.19.** Four poles equally spaced showing the variation of static eccentricity with mechanical position.

The timing change caused by eccentricity can be expressed as a change to the rotational position of the pole / slot interaction and determined trigonometrically by referring to Figure 5.20. The aligned interaction of the pole shown would have occurred at the $\beta^\circ$ position, however, due to the eccentricity, it now occurs $\theta$ degrees earlier (for counter clockwise rotation of the rotor).

To determine the angle $\theta$, the diagonal must first be determined so that it can be used with either the sine or cosine rule to determine the required angle.
5 Implemented Methods for Decoupling Contributions to Cogging Torque

\[
\text{Diag}_\theta = \sqrt{R_{\text{mean}}^2 + Ecc_s^2 - 2 \times R_{\text{mean}} \times Ecc_s \times \cos(180 - \theta_s)} \quad \text{Eq 31}
\]

Where:

\(\theta_s\) is the location of the slot at angle \(\theta\).

\(Ecc_s\) is the maximum static eccentricity being considered

\(R_{\text{mean}}\) is the mean radius

\(\text{Diag}_\theta\) is the diagonal distance between the centre of rotation of the rotor to the actual position of the pole / slot interaction at angle \(\theta\).

Then, using the cosine rule again, the unknown timing change angle, \(\beta_s\), can be determined:

\[
\beta_s = \arccos \left( \frac{R_{\text{mean}}^2 + \text{Diag}_\theta^2 - Ecc_s^2}{2 \times R_{\text{mean}} \times \text{Diag}_\theta} \right) \quad \text{Eq 32}
\]

Where:

\(\beta_s\) is the change in timing angle for a pole / slot interaction at mechanical position \(\theta\).
5.6.2 Assumptions

The static eccentricity analytical method includes two simplifying assumptions:

1. Eccentricity results in the poles overhanging the outer radius of the stator at one particular point and overhanging the inner radius 180° later. In the analytical method, any MMF differences between the pole and the slot resulting from eccentricity are ignored and only the timing alterations...
are considered. Any error induced as a result of this assumption is quantified in section 6.5.1.

2. In addition to the potential MMF changes due to the poles overhanging the stator at the inner or outer radii, there are also slight alterations to the angle of incidence of the pole to the slot due to the eccentricity. This changes the effective skew of the pole relative to the slot as shown in Figure 5.21. The analytical method does not consider this effect on the cogging torque. The error induced with this simplifying assumption is quantified in section 6.5.1.2.

Figure 5.21 Progress of increasing from negative, to aligned, to positive eccentricity misalignment to illustrate the relative skewing of the pole to the slot.

The amount the angle of incidence will be affected will depend on the position of the stator and the direction of the eccentricity. Figure 5.19 demonstrates how the position of the poles relative to the direction of the eccentricity determines the effect on both timing and skew, with poles and slots on the axis in the direction of eccentricity being unaffected. However, poles / slots at 90° to this axis are maximally affected, with the difference in the position of poles 2 and 4 to their respective slots being clear (Figure 5.19).
Figure 5.22 Four poles and four slots at 90° positions to illustrate that while both timing and angle of incidence are affected with eccentricity, the extent depends on the position of the pole / slot with respect to the direction of the eccentricity. In this example, the stator has been shifted to the right and the timing and skew of poles 1 & 3 are unaffected while poles 2 & 4 are affected. The interaction of pole 2 with the slot is advanced while pole 4s interaction is delayed.

5.6.3 Electromagnetic FEA Model

With the introduction of the assumptions listed in the previous section, the same library of waveforms can be used for static eccentricity simulation. The only difference lies with the placement of the slot / pole interactions due to the timing change and that air gap does not vary.

For validation of the analytical / superposition method, a complete motor assembly with static eccentricity was modelled in FEA (Figure 5.23).
5.6.4 Experimental Procedure

Static eccentricity was induced on the experimental test rig (outlined in section 4.4.1) by parallel shifting the rotor across the face of the stator. The rotor was initially offset fully to one side of the stator using the jacking bolts and then adjusted to ensure the rotor and stator faces were parallel. The rotor was progressively shifted from this initial position across the face of the stator, through centre, to the opposite limits of the test rig thus providing +2.75 mm to -0.875 mm of eccentricity data in 0.125 mm increments. By starting at one extreme and moving through centre to the other, the aligned position to within 0.0625 mm was obtained without the need for a highly accurate alignment procedure.

Figure 5.23. Complete motor assembly FEA model with exaggerated static eccentricity for validation of the analytical / superposition method.
5.7 Dynamic Eccentricity

5.7.1 Simulation Method

Simulation of dynamic eccentricity follows a similar procedure to the simulation of static eccentricity described in section 5.6.1 with two main differences:

1. The assembly order of the waveforms. Dynamic eccentricity results when the rotor shaft bore is off centre and as a consequence the rotor spins eccentrically. Thus, the eccentricity of each individual pole does not vary but is unique and each slot sees a sinusoidal variation in eccentricity as the rotor rotates.

2. The torque moment arm for each pole varies and therefore each pole has an increased or decreased magnitude of its cogging torque waveform, depending on whether it is further from or closer to the axis of rotation respectively.

5.7.2 Dynamic Eccentricity Simulation Assumptions

As well as any assumptions listed for static eccentricity, it is assumed that the variation in torque moment arm associated with dynamic eccentricity can be accounted for with a factor applied to the entire pole transition over single slot base waveform. Thus, for a given eccentricity and pole position, the distance from the axis of rotation can be calculated and the base waveform multiplied by the factor. The error induced by this method is quantified in section 6.6.1.
5.7.3 Electromagnetic FEA Model

The FEA data for the library of waveforms to be used with hybrid FEA / analytical method was the same as for the previously discussed misalignments presented in section 5.6.3.

The cogging torque waveform created by superposition was validated by comparing to a complete PMSM FEA model. The complete model appears the same as that presented in the static eccentricity section (5.6.3), however, the centre of the axis of rotation of the rotor was offset so as to simulate dynamic eccentricity. The comparative results are presented in section 6.6.2.

5.7.4 Experimental Procedure

The rotor and stator on the experimental test rig were aligned by analysing the data from the static eccentricity tests and verified by using stainless steel feeler gauges to ensure parallelism. For the dynamic eccentricity, custom fabricated magnet formers were utilised (introduced in section 4.4.2). The custom rotor backing plate was loaded with magnets using one of the laser cut acrylic formers and tested. Each of the 23 magnet formers was installed with the same poles in the same order to ensure the only change made to each rotor was the offset of the poles relative to the centre of the rotor axis, thereby providing data for dynamic eccentricity from 0 to 2.2 mm in 0.1 mm increments.

5.8 Summary – Implemented Methods

As stator and rotor contributions to additional cogging torque harmonics are theoretically independent as shown by other research [18, 27], analysis of all combinations of 10 production rotors and 10 production stators should confirm that rotor affected harmonics do not change with stator number and vice versa.
Therefore, this method can be validated by checking for this independence and then used to determine the relative magnitudes of stator and rotor contributions to overall cogging torque.

Additionally, an analytical method utilising superposition and an FEA developed lookup table was presented to determine the contributions to overall cogging torque from pole misplacement and pole strength variation. The FEA data will be obtained from a library of pole transitions passing over a single stator slot ensuring a complete flux return path. Superposition will then be used to create a production rotor with 20 poles passing 24 slots.

Finally, four types of misalignment conditions possible with axial flux PMSMs can be simulated using a combined FEA and analytical method. These simulation data can be validated with complete FEA models and experimental data obtained from the specially developed experimental test rig.
Chapter 6

Results

Chapter 5 presented the methods used to decouple the rotor, stator, pole strength, pole placement and the four types of misalignment contributions to cogging torque and this chapter presents the results from the implementation of these methods. It is divided into six main sections:

- 6.1 Decoupled stator and rotor contributions
- 6.2 Decoupled pole strength and pole placement contributions
- 6.3 Effects of static angular misalignment
- 6.4 Effects of dynamic angular misalignment
- 6.5 Effects of static eccentricity
- 6.6 Effects of dynamic eccentricity

All sections contain validation of method data and sections 6.2 to 6.6 also contain data to validate any assumptions listed in chapter 5.

Sections 6.3 to 6.6 utilise the methods outlined in chapter 5 to analytically predict the cogging for 10,000 randomly generated PMSMs. The random rotor
and stator combinations were generated with pole and slot placements based on the ranges and standard deviations of the production rotors and stators.

6.1 Decoupled Stator and Rotor Results

This section will initially present results to validate the method implemented to decouple the rotor and stator contributions to cogging torque as outlined in section 5.1, followed by the results from the analysis. Analysis and results presented in this section contributed to the conference paper:

Thiele, M., G. Heins, T. Brown, Decoupling manufacturing sources of cogging torque in fractional pitch PMSM. Electric Machines & Drives Conference (IEMDC), 2011 IEEE International

6.1.1 Validation of Method

As presented in section 5, experimental validation of the implemented method was required to confirm that stator and rotor harmonics are independent and do not interact. To achieve this, the cogging torque from all possible combinations of 10 production rotors and 10 stators were measured. The magnitude of stator affected harmonics should be constant for all combinations of rotor and, likewise, the magnitude of rotor affected harmonics should be invariant with stator.

Figure 6.1 shows the 24\textsuperscript{th} order harmonic plotted as a function of both rotor number and stator number. It highlights the dependence of the magnitude of the 24\textsuperscript{th} with rotor number as there is limited variation in cogging torque for a given rotor, irrespective of the stator with which it was tested. That is, the magnitude of the 24\textsuperscript{th} order harmonics is invariant with stator number. This can be further confirmed by considering the standard deviation across the rotor tests.
and stator tests. Standard deviation with changing stators for the 24\textsuperscript{th} harmonic was 0.096 while across rotors it was 0.395, or 4 times higher (Figure 6.2).

Figure 6.1  Surface mesh of RMS cogging torque of 24\textsuperscript{th} order harmonics as a percentage of rated torque (Z axis), Rotor number (Y axis) and Stator number (X axis). Shows that the 24\textsuperscript{th} order varies with rotor number but is consistent with stator number. That is, the 24\textsuperscript{th} order of the cogging torque is not affected by different stators and is therefore a function predominantly of the rotor.
Rotor 3 stator 10 appears to be an outlier that is somewhat inconsistent with the remainder of the data. This is visible in Figure 6.1 where the cogging torque of the 24th harmonic for rotor 3 is significantly higher for stator 10 than stators 1 to 9. This is also confirmed in Figure 6.2 where the standard deviation within rotor 3 is much larger than for all other rotors. One possible explanation for the data is that the rotor 3 stator 10 test may have been affected by variations in air gap caused by incorrect torqueing down of the slider platform on the cogging torque test rig. If this hypothesis is correct, the data should show an increase in RMS cogging torque for all harmonics compared with neighbouring tests, not just for the 24th harmonic. This is shown in Figure 6.3 where the normalised RMS cogging torque is plotted for each of the 100 combinations of 10 rotors and 10 stators. Rotor 3, stator 10 does not follow the trend of being affected by the stator in the same way as the other tests.
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Figure 6.3. Normalised RMS cogging torque as a percentage of rated for each rotor and stator combination showing that Rotor 3, Stator 10 is affected across all frequencies, not just the 24th, thus, a smaller air gap could have been the possible cause.

Figure 6.4 shows the 20th order harmonics as a function of rotor number and stator number and confirms the independence of the rotor and the dependence of the stator to the 20th order harmonic cogging torque as seen by the straight line corresponding to each stator. The standard deviation with changing rotors was 0.056 and changing stators was 0.217, or 3.8 times higher (Figure 6.5).
Figure 6.4 Surface mesh of RMS percentage cogging torque for 20th order harmonics (Z axis), Rotor number (Y axis) and Stator number (X axis). Shows that the 20th order varies with stator number but is consistent with rotor number. That is, the 20th order of the cogging torque is not affected by different rotors and is therefore a function predominantly of the stator.
6.1.2 Decoupled Stator and Rotor Results

The method discussed in section 5.1 was applied to the experimentally derived cogging torque waveforms from all combinations of 10 production stators and 10 production rotors. The results for the contribution to total additional cogging torque from the stator, rotor, native and ‘other’ are shown in Figure 6.6. Figure 6.7 shows that, on average, 35.8% of the RMS cogging is due to imperfections within the rotor, 29.8% the stator, 6.0% is the native or design component and 28.3% is due to other causes such as static and dynamic eccentricity and static and dynamic angular misalignment, the results of which will be presented and discussed in sections 6.3 to 6.6 of this chapter. There is less variation of the cogging torque caused by the stators than that of the rotors.
Figure 6.6. Total, rotor, stator and native contributions to overall cogging torque. What is not due to stator, rotor and native is categorised as ‘Other Cause’ and includes misalignment. Centre ‘dot’ is the average with ±1 standard deviation.
6.1.3 Discussion of Decoupled Stator and Rotor Results

Experimental testing of all 100 combinations of 10 production rotors and stators confirmed the independence of rotor and stator affected harmonics. This method was used to quantify the contributions to additional cogging torque for which each of these components is responsible. On average, the rotor was responsible for 36% and the stator 29% of the PMSM cogging torque.

One of the experimental tests displayed a higher than expected cogging torque and data was presented to support the hypothesis that a reduced air gap was responsible for the non-conforming data.

Figure 6.7. Average cogging torque sources as a percentage of total cogging torque.
6.2 Decoupled Rotor Pole Strength and Placement Results

In section 6.1, the results for the individual contribution to cogging torque of the rotor and stator were presented. This section will present the results of the rotor contributions to cogging torque, specifically, the effect of pole placement inaccuracy and pole strength variation to the PMSM cogging torque additional harmonics.

Initially, however, the results for the validation of assumptions outlined in section 3.1 and 5.2.1 will be presented, followed by the validation of the method that was presented in section 5.2.3. Analysis and results presented in this section contributed to the conference paper:


6.2.1 Validation of Assumptions – Rotor Decoupling

Chapter 5.2 presented the method for decoupling the pole strength and position which included several assumptions. These were:

- Standard superposition assumptions – linearity (section 3.1)
- Pole strength variation results only in alterations to the scale of the cogging torque waveform
- Pole skew variation has little or no effect on cogging torque waveform
- Magnetic saturation is not critical for the rotors tested
- A pole transition, that is, two poles passing a single stator slot can be used as the foundation waveform to create complete motor cogging torque waveforms using superposition.
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The validation of each of these assumptions will now be addressed.

6.2.1.1 Validation of Assumption - Pole Strength Only Affects the Scale of the Cogging Torque

The superposition based analytical method relies on the ability of pole strength to be varied through scale adjustments. Therefore, the method’s accuracy is inherently related to the accuracy of a weak pole being made to appear stronger and vice versa through scale adjustment only.

The error inherent in this assumption was validated both experimentally and using FEA. The experimental method involved testing 20 individual poles past a single slot stator, creating 20 separate waveforms, all with unique cogging torque magnitudes due to the variations in pole strengths (Figure 6.8 top pane). These waveforms were then normalised (Figure 6.8 middle pane) and the error between each waveform and the mean determined as a function of angle (Figure 6.8 lower pane). Experimentally, the maximum error induced through scaling was found to be 2.5%, with the majority of poles falling within the range of ±2%.
Figure 6.8. Cogging torque waveforms for 20 poles tested one at a time past the single slot stator (top pane). Middle pane, normalised cogging torque and lower pane, error induced from the scaling, with a maximum of ±2.5%, and most falling within the ±2% range.
Similarly, a pole transition with 3 different magnet strengths was modelled in FEA, one at normal N35 strength (890,000 A/m) and one each at ±10% of N35 standard strength. The three individual cogging torque waveforms are shown in the top pane of Figure 6.9. The middle pane of the same figure shows the standard strength N35 pole scaled to match both the strongest and weakest poles, with the lower pane showing the induced error. It is expected that most of this error is actually from FEA approximations rather than error within the assumption. This theory is supported by the fact that the zero torque points for all 3 magnets are common, indicating that there is no frequency modulation associated with the change in pole strength, only scaling.
Figure 6.9 Pole transition cogging torque with 3 variations in pole strength (top pane), middle pane, the ±10% strength variation poles are scaled to match the normal strength N35 pole and, lower pane, the induced error.
6.2.1.2 Validation of Assumption – Rotor Magnetic Saturation

Previous research [86] indicated that the use of a single pole when using superposition resulted in a decrease of the accuracy of the method due to a lowering of the flux density associated with the removal of the flux return path and/or saturation of the rotor backing plate. It was for this reason that a pole transition of two poles was used for the generation of the superposition waveform.

As the fundamental superposition waveform uses two poles to recreate a full 20 pole waveform, previous research [86] has suggested a possible source of error as magnetic saturation. This results from the 20 poles potentially saturating the rotor backing plate and the magnetic flux density per pole decreasing as a consequence. Therefore, when a pole transition waveform is used, the flux density may not be attenuated by saturation which would result in an overestimation of the cogging torque when using superposition.

To determine the effect of saturation, the magnitude of the FEA derived cogging torque waveform from a two pole transition over a single stator slot was compared to the waveform from a complete rotor (20 poles) over a single slot. These data are shown in Figure 6.11 and confirms that magnetic saturation in this motor has limited effect and is therefore not a significant source of error. This could be due to the rotor plate thickness being designed to ensure saturation did not occur for maximum flux density.
Figure 6.10 Left, 20 pole 1 slot FEA model and right, 2 pole 1 slot FEA model used to simulate the data presented in Figure 6.11.
Effect of Number of Poles on Cogging

Saturation Validation, Time Domain

Saturation Validation, Frequency Domain

Figure 6.11. Cogging torque waveform for a pole transition as part of a complete rotor (blue) compared to a pole transition of 2 poles (green) (pane 1) and the error as a percentage of the maximum cogging torque (pane 2). Panes 3 & 4 are the same data in the frequency domain.
6.2.1.3 Validation of Assumption - Pole Skew Variation Has Limited Effect

For the motors tested, the placement of the 20 poles was controlled by a plastic magnet retainer, however, the fit of the poles varied and some were able to be slightly rotated within the confines of the retainer due to a combination of small variations in magnet size and variation in the size of the magnet pockets in the magnet retainer (Figure 6.12 and Figure 6.13), marginally affecting the skew of the magnet. It was therefore necessary to determine if this variation in skew would result in changes to the magnitude of the 24th order harmonics, as this would imply that the 24th order could be affected by not only pole location and strength, but also by skew, which would necessitate further decoupling.

To maximise the variation in skew, the combination of a small magnet and a large magnet retainer pocket (representing the extremes of the realistic manufacturing induced error) were tested using the single slot stator. The single pole cogging torque with the magnet in a fully clockwise (CW) and counter-clockwise (CCW) position was measured experimentally.

Figure 6.14 shows the difference between the cogging torque profiles of the same magnet in the two skewed positions while Figure 6.15 shows the frequency spectrum for the two waveforms. The maximum error induced is within -1% to 2% in the time domain which demonstrates the minimal effect the worst possible skew combination has on cogging torque. Therefore, skew variation can be disregarded for this analysis.
Figure 6.12. Pole shown skewed fully counter clockwise in magnet retainer. Red circle highlights the gap on the left side of the pole.

Figure 6.13. Pole shown skewed fully clockwise in magnet retainer. Red circle highlights the gap on the right side of the pole.
Figure 6.14. Effect of pole rotation on cogging torque. CW is the cogging torque from the magnet rotated as far as possible in the magnet holder in the clockwise direction (Figure 6.13) and CCW is the pole rotated in the counter clockwise position (Figure 6.12). Top pane is time domain cogging torque, lower pane is the percentage error between the two skews in the time domain.
Figure 6.15. Effect of pole rotation on cogging torque in the frequency domain. The maximum percentage error for the critical orders (18 – 30) is less than 0.35%.

6.2.2 Validation of Method

Section 5.2 presented the method used to decouple the pole strength and placement contributions to cogging torque. Two approaches were used to validate this method:

1. Experimental data from 40 custom manufactured rotors with known pole position errors were tested with the single slot stator and the position of the poles detected using the pole over slot zero torque point.

2. Two FEA rotors were modelled, both with known pole position errors, but one had uniform pole strength while the other had ±10% pole strength variation (random normally distributed). The locations of the pole were detected and compared to the known locations.
The results from the validation testing and analysis are presented in the next two sections.

### 6.2.2.1 Experimental Validation Using Custom Manufactured Rotors

The pole position method was validated by testing 40 custom manufactured rotors with error induced in the positioning of the 20 poles. The poles were 10 mm diameter round disk magnets to avoid any effect of magnet skew, positioned by a laser cut acrylic former (Figure 6.16). The placement error induced was within the range of ±1.8° and was randomly distributed. The rotors cogging torques were analysed and pole positions determined. These data were compared to the known data for accuracy, with Figure 6.17 showing the comparison between the known and detected pole locations for one rotor. It is not feasible to display the data from all 40 rotors in this manner, and the remainder has been presented in a histogram in Figure 6.18.
Figure 6.17  Pole detection accuracy for one of the 40 custom manufactured rotors with round poles positioned to known but random locations. Top pane shows the known location of the poles (red squares) and the detected locations (blue crosses). Lower pane is the difference between the two.

The average position accuracy of the poles was 0.13° with a standard deviation of 0.1° (Figure 6.18), although these data also include the unknown inaccuracy from the CNC laser cutting of the acrylic magnet former in addition to any possible measurement system inaccuracy. The fit of the magnets to the acrylic
former was ‘snug’ and was not expected to directly contribute to any measurement inaccuracy.

Figure 6.18  Histogram of pole deviation from intended position for the 40 custom manufactured rotors (for validation of method only).

6.2.2.2 FEA Validation

The analytical method was also validated using two FEA models, both of which consisted of a 20 pole rotor and a single slot stator. For one or the rotors, the poles were uniform N35 strength while the other had variable pole strength
(N35±10% of the standard strength within a random normal distribution as shown in Figure 6.19). The FEA models were constructed with a 0.2% energy error mesh and the cogging torque determined every 0.5 degrees. These waveforms were then cubic spline interpolated in Matlab to increase the resolution to 0.05°, thereby increasing the possible accuracy for the detection of the zero torque point to ±0.025°.

Figure 6.19 Pole coercivity (A/m) for the FEA rotor with variable strengths (blue circles), with the mean value shown in dashed blue and a standard N35 magnet strength shown in green dashed.

The cogging torques were analysed and the apparent pole locations determined from the pole over slot zero torque point and compared to the known locations of the poles. Figure 6.20 is a section of the cogging torque for both rotors and shows that while the position of the zero torque point created when the gap
between the pole transition is over the stator slot does vary with magnet strength, the pole over slot zero torque point is comparatively stable.

Figure 6.20 FEA cogging torque of 9 of the 20 randomly placed poles passed a single slot stator, with uniform pole strength (blue) and ±10% strength variation (random normal distributed) (green). The absolute pole positions are indicated in red.

The location of the poles for both FEA models was determined using the pole over slot zero torque point and compared to the known pole locations. Figure 6.21 shows the detected pole location deviation from the known location, with the red squares representing the known locations as modelled in FEA, the blue
crosses being the detected locations of the constant strength pole waveform and the green circles the detected locations of poles with variable strengths. The RMS error (degrees) for both the uniform and variable pole strength rotors was 0.098° indicating that pole strength variation did not affect the average accuracy of the pole detection. All but 5 of the variable strength poles were detected in the same locations as the constant strength poles as shown in the lower pane of Figure 6.22. This validates that pole strength does not adversely affect the position of the pole over slot zero torque point and that it can be used to detect pole location with an average accuracy of less than 0.1°.
There are two reasons for the differences between the detected locations of the 5 uniform strength and variable strength poles. Firstly, the minimum resolution of the detection is 0.05° and this is dictated by the number of data points in each of the cogging torque waveforms (7200 in this case). The other is due to inaccuracies in the FEA data. Figure 6.22 shows one of the pole locations with the two FEA cogging torque waveforms (blue is constant strength and green is variable strength). The undulations, particularly in the variable strength...
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waveform, are a result of the accuracy of the FEA raw data and cubic spline interpolation process. Further improvements in accuracy could be achieved with higher resolution sampling within FEA, say every 0.1° rather than 0.5° and also reducing the energy error of the mesh, although both of these would greatly increase the computation time and this is the reason they were not implemented (the mesh at 0.2% energy error and sampling every 0.5° required 112 hours of processing per rotor on a 8 core, 3.2GHz PC with 32GB of ram running a 64 bit operating system).
Figure 6.22 FEA inaccuracy is the probable cause of the difference between the detected locations of the poles with uniform and variable strength. The undulations on the green line, the variable magnet strength cogging torque, shifts the zero torque point.

6.2.2.3 Summary of Validation of Method for Pole Strength and Placement Decoupling

This section has validated the following assumptions and methods:

- Pole strength can be adjusted by scale adjustment with less than ±2% error
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- For the research motor, reducing the number of poles does not alter flux density of a pole transition by more than ±2%.
- Pole skew variation within the confines of a production pole former made less than ±2% difference to the cogging torque.
- Pole locations can be detected to within ±0.2° by using the pole over single slot zero torque point.

6.2.3 Rotor Contributions to Cogging Torque Results

Section 6.2.1 presented the results to validate the assumptions relating to the implemented method and section 6.2.2 presented the results validating the actual method itself. This section presents the pole strength and position results from the 10 rotors tested and their contribution to motor cogging torque.

6.2.3.1 Pole Position Results

The pole positions were determined using the pole over slot zero cogging torque point method outlined in section 5.2.3. The pole offsets for the production rotors were determined and found to fall within the range of ±0.5° with the majority of poles placed to within ±0.25° (Figure 6.23).
6.2.3.2 Pole Strength Results

The pole strengths were determined using the superposition and least squares minimisation method outlined in section 5.2.4.

The majority of poles were found to fall within the range of ±4% of the mean strength, with all the poles being within ±8% (Figure 6.24). As pole strength is a function of both magnet coercivity and magnet thickness, the determination of...
the true root cause of the strength variation would require measurement of the magnet thickness.

![Histogram of percentage pole strength variation from the mean.](image)

**Figure 6.24** Histogram of percentage pole strength variation from the mean.

6.2.3.3 Pole Strength and Position Contributions to Motor Cogging Torque.

The least squares minimisation and analytical methods described in section 5.2.4 were used to create waveforms representing:

1. Production rotors with strength variation and pole misplacement
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2. Production rotors without pole strength variation
3. Production rotors without pole placement error
4. Production rotors perfect placement and uniform pole strength

The RMS cogging torque for each of the four configurations was determined with the results presented in Figure 6.25. It shows that uniform pole strength would reduce rotor contributions to cogging torque by 14% and as the rotor is responsible for 35.8% of motor additional cogging torque harmonics, this would reduce motor cogging torque by approximately 5%. Perfect pole placement would reduce rotor contributions to cogging torque by 58% and therefore the motor cogging torque by 21%. Both perfect pole placement and uniform pole strength would reduce rotor contributions by 70% which is a 25% reduction for the motor. These data are summarised in Table 3.

Please note that the 10 production rotors upon which these data are based are not the same rotors used for the stator and rotor decoupling in section 6.1, hence the total rotor contributions are not aligned.
Figure 6.25. The cogging torque that could be achieved, on average, if only pole strength variation was eliminated (Perfect Strength), poles with strength variation were positioned perfectly (Perfect Position) and poles of uniform strength were positioned perfectly (Both Perfect). Error bars with ±1 standard deviation, experimental data from 10 production rotors.
Figure 6.26 Percentage reduction in cogging torque possible with only uniform pole strength, only perfect pole placement and both perfect placement and uniform strength.

Table 3 Summary of rotor contributions to cogging torque.

<table>
<thead>
<tr>
<th>Error</th>
<th>Rotor contributions</th>
<th>Motor contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole strength variation</td>
<td>14%</td>
<td>5%</td>
</tr>
<tr>
<td>Pole placement error</td>
<td>58%</td>
<td>21%</td>
</tr>
<tr>
<td>Both strength and placement</td>
<td>70%</td>
<td>25%</td>
</tr>
</tbody>
</table>
6.2.4 Discussion of Pole Strength and Placement Results

This section detailed the results for the decoupling of the rotor contributions to additional cogging torque. The pole placement of the 10 production rotors was determined by detecting the pole over single slot zero torque point. Once the locations were known, pole transitions of the same spacings were assembled using superposition to create a cogging torque waveform with uniform pole strength. A least squares minimisation was then performed to determine the pole strength variation.

The developed method was able to determine strength and location without the need of a Gauss meter which utilise specialised, delicate and expensive Hall effects probes that are thin enough to be inserted in the air gap between the poles and stator teeth. A robust alternative to using a Gauss meter which may be more suitable for manufacturers could be to obtain pole position and strength tolerance data online by rotating the rotors passed a single slot stator and measuring the cogging torque.

6.3 Static Angular Misalignment Results

Static angular misalignment was defined in section 2.1.7.3 and can be caused by inaccurate machining of the bearing bore or the stator being at an incorrect angle. This section initially validates the assumptions implemented with the analytical method used to predict the effect of static angular misalignment on cogging torque. Subsequently, the analytical method is validated with FEA data, then the superposition results are presented and finally the experimental results.
6.3.1 Validation of Assumptions

6.3.1.1 Assumption that pole angle has minimal effect

The assumption listed in section 5.4.2 stated that the air gap variations caused by the angular misalignment of the pole / slot interactions could be substituted by parallel pole / slot interactions with an equivalent mean radius air gap. FEA derived results of three configurations are presented to quantify the error induced with this assumption.

The three FEA models were all a pole transition over a single slot with the following configurations:

1. The first was a standard configuration – 1 mm parallel air gap as shown in Figure 6.27.
2. The second had a 1 mm mean radius air gap but was also angled to the stator by 0.8° therefore the inner edge of the pole had a smaller air gap and the outer edge had a larger air gap as shown in Figure 6.28.
3. The third had a 1 mm mean radius air gap but was also angled to the stator by -0.8° therefore the inner edge of the pole had a larger air gap and the outer edge had a smaller air gap as shown in Figure 6.29.
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Figure 6.27 Parallel pole transition and single slot to provide the reference waveform.

Figure 6.28 Exaggerated angle of the stator with positive inclination effectively increasing the air gap. Note, the air gap was adjusted to be 1 mm at the mean radius.
Figure 6.29  Exaggerated angle of the stator with negative inclination effectively decreasing the air gap. Note, the air gap was adjusted to be 1 mm at the mean radius.

The error induced with a 0.8° inclination of the stator was a maximum of ±5% (Figure 6.30 and Figure 6.31) with a RMS of approximately 2.5%. The reason for the difference is related to the location of the reduced air gap. For inclinations which tend to reduce the effective air gap, the majority of the reduction occurs at the outside radius of the magnets which has two effects:

1. The torque moment arm to the smallest air gap of the poles is larger which increases the magnitude of the cogging torque and
2. The pole arc to slot width ratio is increased due to the skewed trapezoidal pole shape thereby increasing the duration of the pole transition and slot interaction.
Figure 6.30 Cogging torque for the parallel and +0.8° of stator inclination (top pane) which increases the effective air gap. The centre pane shows the parallel and -0.8° degree inclination pole transition over slot waveforms. Lower pane is the percentage error of the angled waveforms compared to the parallel.
Figure 6.31  Frequency domain cogging torque for the +0.8° of stator inclination and a parallel pole transitions with equivalent mean radius air gap (top pane). The centre pane shows the parallel and -0.8° degree inclination pole transitions over slot frequency waveforms. Lower pane is the percentage error between the two.
6.3.1.2 Determination of slot and pole placement standard deviations for hybrid FEA / analytical model

Using the hybrid FEA / analytical method to create random PMSM cogging torque waveforms, a known standard deviation for both pole and slot placement is required. The results in section 6.2.3.1 showed that pole placement was normally distributed within the range of ±0.8° from the ideal position (18° for the motor investigated). Additionally, analysis of 16 production stators tested with a single pole to allow the determination of the slot locations showed that the stator slots were distributed between 14.5° and 15.5° degrees. Figure 6.32 shows the cogging torque waveform for one of the stators with the stator slot locations shown in red squares. Figure 6.33 shows the histogram for all slot locations.
Figure 6.32 Cogging torque of a single magnet passing a complete stator. Red squares are the zero torque points corresponding to the tooth locations.
6.3.2 Validation of Method

To validate that the static angular misalignment analytical method was capable of predicting the effect of static angular misalignment, a complete 20 pole, 24 slot PMSM with 0.6° of static angular misalignment was modelled in FEA and the cogging torque in the time and frequency domain was compared to that predicted by the same PMSM configuration but modelled using the hybrid FEA / analytical method (Figure 6.34). The data suggests that the maximum error
expected in the time domain is predominantly within ±2% of rated torque, although this is dependent on the correct alignment of the two waveforms in the position domain. In the frequency domain, the maximum error is less than ±1.5% for the critical orders, suggesting that the analytical method is capable of predicting PMSM cogging torque with imperfect pole and slot placement as well as angular misalignment.
Figure 6.34 Validation of the analytical method used to simulate static angular misalignment. Top pane shows FEA predicted (blue) and Superposition predicted (green) with the error between the two as a percentage of rated torque in pane 2. Pane 3 is the FFT of both the FEA and Superposition cogging torque with the error between the two in pane 4.
6.3.3 Analytical Results

The cogging torque for 10,000 rotors and stators with random pole and slot placement and five levels of static angular misalignment from 0° to 0.8° in 0.2° increments were modelled to show the relationship between cogging torque and static angular misalignment, pole placement accuracy and slot placement accuracy. Figure 6.35 shows the effect of increasing magnitudes of static angular misalignment and that misalignment predominantly affects the magnitude of the 23rd and 25th harmonics (1st order sidebands of the 24th harmonic). In addition, the rotor and stator inaccuracies do not have a major influence on the effect of the misalignment. Therefore, the presence of 23rd and or 25th harmonics (or 1st order sidebands of the rotor affected harmonic for the general case) could be used as an indicator that an axial flux PMSM is assembled with static angular misalignment.

Figure 6.35 also shows, however, that it is difficult to infer the exact magnitude of the static angular misalignment from a given sideband magnitude. For example, if the 1st order sideband of the 24th harmonic was 10 mNm, the misalignment could be anywhere from approximately 0.5° (where the 1st blue dashed line is crossed) to 0.8° with 68% accuracy. To contain 95% of the data, the misalignment could be anywhere from 0.3° of angular misalignment up to the maximum possible. Thus, sideband magnitude is multi-dependent on angular misalignment and slot and pole placement accuracy.

Note that 0.4° was the maximum misalignment possible on the experimental test rig.
Figure 6.35  Superposition results for 10,000 randomly generated PMSMs with increasing static angular misalignment. First order sidebands about the 24th harmonic, on average, increase with increasing misalignment. Red lines contain 95% of data and blue dashed lines 68%. 
6.3.4 Experimental Results

The experimental static angular misalignment data was obtained by placing shims of varying thicknesses behind the stator as described in section 5.4.4. Figure 6.36 shows the presence of first order sidebands about the 24\textsuperscript{th} harmonic as was predicted by the analytical model, however, the magnitude of the experimental data is significantly higher than that predicted. This discrepancy is addressed in detail in the next section, 6.3.5.
Figure 6.36 Experimental results for static angular misalignment. Data obtained by placing shims behind the stator to induce varying levels of misalignment. Green circles are experimental data, solid red lines contain 95% of analytical data and dashed blue lines 68%.
6.3.5 Discussion of Static Angular Misalignment Results

The analytical simulation for static angular misalignment with both random pole and slot placement revealed that as misalignment error increased, so too did the first order sidebands of the 24\textsuperscript{th} harmonic. For the general case this would be the first order sidebands of the rotor affected harmonic (the harmonic corresponding to the number of stator slots, \( N_s \)). The experimental data responded in a similar manner in that the 24\textsuperscript{th} order sidebands increased with increasing static angular misalignment, however, there is a significant offset compared to the analytical data. This is highlighted by comparing the aligned data, where the analytical predicted sideband magnitude is less than 10 mNm but the experimental test data is 23 mNm. There are two possible reasons for the difference:

1. There is some inherent static angular misalignment built into the experimental test rig. Chapter 4 highlighted that the twin shaft design had both advantages and disadvantages, with the largest disadvantage being that rotor to stator alignment was not inherent. While significant effort was invested in aligning the two shafts prior to testing, the only alignment possible is in the horizontal plane; vertical plane alignment is built into the test rig and adjustment is not possible. Thus it is possible that static angular misalignment in the vertical plane is responsible for some of the 24\textsuperscript{th} order sidebands. However, comparing the magnitude of the experimental 24\textsuperscript{th} order sidebands to those predicted by the analytical method, the magnitude of the misalignment should be in the order of 1°. A 1° misalignment between the rotor and stator would be induced by a 1.7 mm difference in the air gap from one side of the motor to the other. The rotor to stator alignment of the test motor was adjusted with stainless steel feeler gauges to a parallelism within ±0.1 mm and therefore misalignment would not be expected to account for the full magnitude of the 24\textsuperscript{th} order sidebands.
2. The other possible cause could be due to a characteristic that exists in the production motors which is not accounted for in the analytical model. Analysis of ten production rotors revealed that pole placement was normally distributed (section 6.2.3.1), however, analysis of 16 stators tested with one magnet reveals that there is a significant 1st order amplitude modulation of the resulting cogging torque (sample stator shown in Figure 6.37), most likely from the ‘punch and wind’ manufacturing process responsible for the production of the wound toroid stator. Some of the 16 stators have a significant first order component in the deviation from the ideal location of the stator slots as can be seen in Figure 6.38. The systematic (non-random) misplacement of the slots causes a variation in the stators reluctance which in turn causes a first order amplitude modulation of the cogging torque. When static angular misalignment is present, the cogging torque also first order amplitude modulates, but this is due to the variation in air gap rather than any direct change to the stator reluctance. Figure 6.39 shows the analytically determined magnitude of the 24th order sidebands that result from the combination of 16 production replicated stators and 100 rotors with normally distributed random pole placement. Generally, as the magnitude of the first order slot placement increases, so too does the magnitude of the 24th order sidebands, however, when coupled with a rotor that has accurate pole placement, the sideband magnitude is reduced. Similarly, a rotor with a large pole placement standard deviation when coupled with an accurate stator also does not induce significant 24th order sidebands.

Given the aforementioned, it is not possible to differentiate if 24th order sidebands are caused by static angular misalignment or first order stator slot misplacement, other than to do a sweep of varying levels of static angular misalignment which would cause the 24th order sidebands to pass
through a minimum when most closely aligned. Any 24\textsuperscript{th} order sidebands remaining at that point could be due to stator slot misplacement.

This highlights the importance of experimental data in capturing the stochastic nature of manufacturing processes, as FEA or analytical methods alone would not have identified the stator manufacturing errors that contributed to the 24\textsuperscript{th} order sidebands.
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Figure 6.37  Cogging torque of one of the 16 stators with 1 pole / 24 slots showing the significant 1\textsuperscript{st} order amplitude modulation in the stator in the time (pane 1) and frequency domains (pane 2). The 4\textsuperscript{th} order is most likely from the 4 drilled and threaded holes in the rear of the stator which are used for mounting the stator in the motor.
Figure 6.38  FFT of stator slot location showing some stators have a significant 1st order most likely due to the method of manufacturing the stator.
Figure 6.39 Analytical results of the effect of pole placement accuracy and stator slot 1\textsuperscript{st} order magnitude on the magnitude of 24\textsuperscript{th} order sidebands. Slot placement from 16 production stators. Pole placement normally random distributed. It shows that a large first order of slot placement can result in a large 24\textsuperscript{th} order magnitude, if the rotor pole placement standard deviation is also large, however, a small stator slot first order magnitude always results in small 24\textsuperscript{th} order sidebands.

This can also be seen in data presented in subsequent sections for the other misalignment types, particularly dynamic angular misalignment (section 6.4).

While there is an offset to the relevant sideband data, there is a similar gradient to the analytical data suggesting that the induced angular misalignment adds to that inherent in the test rig to further increase the sidebands. This is shown more clearly in Figure 6.40 where the effect of the offset has been removed.
6.4 Dynamic Angular Misalignment Results

As the applicable assumptions for the generation of dynamic angular misalignment superposition data are the same as those previously validated for static angular misalignment in section 6.3, this section has no assumptions to validate. This section validates the method then presents the hybrid FEA /
analytically predicted dynamic angular results followed by the experimentally derived results.

6.4.1 Validation of Method

As was the case for static angular misalignment, dynamic angular misalignment validation is determined by comparing a hybrid FEA / analytically predicted cogging torque waveform in the time and frequency domains to that predicted from FEA with an identical configuration. Thus, 0.6° of dynamic angular misalignment with known imperfect placement of the poles and slots was modelled in FEA and the cogging torque determined for a complete revolution. Figure 6.41 shows the maximum error in the time domain is within ±2% of rated torque while in the frequency domain the error is -0.2% to 0.4% of rated torque, particularly in the critical order region from 18 to 28 orders.
Figure 6.41 Validation of analytical method for simulating Dynamic Angular Misalignment. Top pane shows FEA predicted (blue) and Superposition predicted (green) with the error between the two as a percentage of rated torque in pane 2. Pane 3 is the FFT of both the FEA and Superposition cogging torque with the error between the two in pane 4.
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6.4.2 Analytical Results

The cogging torque for the 10,000 randomly generated rotor / stator combinations previously analysed was determined using the hybrid FEA / analytical method outlined in section 5.5.1. The cogging torque for dynamic angles from 0 to 0.8° in 0.2° increments were determined and the results are shown in Figure 6.42. It reveals that the first order sidebands of the stator affected harmonic (20th order for the research motor) increase in a near linear relationship with increasing angular misalignment from an average of 3 mNm in the aligned configuration to an average of 24.5 mNm with 0.8° of dynamic angular misalignment. The other harmonics and sidebands are negligibly affected.
Figure 6.42 Analytical results for 10,000 randomly generated PMSMs each with 5 levels of increasing dynamic angular misalignment. First order sidebands about the 20th harmonic, on average, increase with increasing misalignment. Red lines contain 95% of data and blue dashed lines 68%.
6.4.3 Experimental Results

These data were obtained by placing shims of various thicknesses behind the rotor backing plate as described in section 5.5.3. Figure 6.43 shows that first order sidebands of the 20th harmonic are a reasonable indicator of the degree of dynamic angular misalignment which is in agreement with the superposition data from the previous section. Clearly the experimental data is noisier than the superposition data highlighting the non-ideal test conditions and indicating that some degree of mixed misalignment may have been present.
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Dynamic Angular Misalignment, 10,000 Random PMSMs, 20th & 24th

Figure 6.43 Experimental results for dynamic angular misalignment. Data obtained by placing shims behind the rotor to induce varying levels of misalignment. First order sidebands of the 20th harmonic correlates with the superposition data. Green circles are experimental data while the solid red lines contain 95% of superposition data and blue dashed lines 68%.
6.4.4 Discussion of Dynamic Angular Misalignment Results

The analytical method predicted that increasing levels of dynamic angular misalignment would increase the 1st order sidebands of the 20th harmonic with other harmonics only minimally affected. The experimental data supported this albeit with first order sidebands of the 24th that were significantly larger than those predicted by the analytical procedure. The reasons for the large sidebands of the 24th are due to the 1st order misplacement of the stator slots as discussed in section 6.3.5.

6.5 Static Eccentricity Results

In section 5.6 the method to create the analytical data was presented along with the assumptions necessary for its implementation. This section initially validates the listed assumptions and then the method. The analytical and experimental results are then presented. The analysis conducted and results presented in this section contributed to the conference paper:


6.5.1 Validation of Assumptions

As discussed in section 5.6.2 there are two assumptions associated with the hybrid FEA / analytical method for predicting PMSM cogging torque with static eccentricity. These are:
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1. Poles overhanging the outer or inner radii of the stator due to the eccentricity have minimal effect on the cogging torque of a pole transition over a single slot.

2. Changing the angle of the pole / slot interaction due to eccentricity has minimal effect.

The quantification of these two assumptions is now considered.

6.5.1.1 Validation of Pole Overhang Assumptions

Static eccentricity within the range of ±2 mm (Figure 6.44 and Figure 6.45) was found to have limited impact on the FEA derived pole transition over single stator slot cogging torque waveform (Figure 6.46), with the eccentric data being within ±2% of the aligned data in the time domain and -0.5% to +1.5% in the frequency domain for the critical orders. Negative eccentricity conditions where the poles are overhung from the stator as shown in Figure 6.44 were slightly more erroneous than the positive eccentricity condition (Figure 6.45) most likely due to fringing effects being exacerbated by the combination of a larger pole arc at the pole outer radius and that these effects are occurring at the maximum torque arm.
Figure 6.44 Poles overhung from stator outer radius (-ve static eccentricity), exaggerated for clarity.

Figure 6.45 Poles overhung from stator inner radius (+ve static eccentricity), exaggerated for clarity.
Figure 6.46 Effect of static eccentricity on the cogging torque of the pole transition over single stator slot is limited.
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6.5.1.2 Validation of Skew Alignment Assumptions

Section 5.6.2 highlighted that eccentricity altered the pole skew relative to the slot and also stated that one of the simplifying assumptions of the analytical method would be to disregard this effect. This section will quantify the effect of altered skew on cogging torque and thereby quantify the possible error induced with this assumption.

The effect of the skew alteration on the cogging torque is shown below in Figure 6.47, which shows a substantial difference between both the phase and amplitude for cogging torque waveforms with -2 mm, 0 mm and +2 mm of eccentricity. In the time and frequency domains, the error between the eccentric cogging torques and the aligned is in the range of ±10%. However, this is the maximum error condition and only occurs at pole / slot positions that are 90° and 270° to the direction of the eccentricity, with no effect at the 0° and 180° positions and a sinusoidal variation in between these maxima and minima. When the average condition is modelled, such as when the eccentricity is directed at an angle of 45° to the pole / slot in question, the error reduces to -3% to +5% as shown in Figure 6.48.
Figure 6.47 Maximum error induced by alteration of pole to slot skew due to the effect of eccentricity for a pole transition over a single slot. The maximum error is induced at the pole / slot location that is 90° to the direction of the eccentricity.
Figure 6.48 Error resulting from additional skew due to the effect of eccentricity at 45° to the pole transition over a single slot.
6.5.2 Validation of Method

As was the case for both static and dynamic angular misalignment, validation of the hybrid FEA / analytical method for static eccentricity is confirmed by comparing cogging torque generated from superposition to FEA for a PMSM with identical pole and slot placement and 2 mm of static eccentricity. Figure 6.49 shows that the analytical method simulated cogging torque falls within ±3% of rated torque in the time domain and ±1.5% in the frequency domain.
Validation of Method - Static Eccentricity Misalignment

**Time Domain**

![Cogging Torque vs Mechanical Position](image)

**Frequency Domain**

![Cogging Torque vs Orders](image)

**Frequency Domain Error**

![% Error of Rated Torque vs Orders](image)

Figure 6.49 Validation of method for 2 mm of Static Eccentricity. Top pane shows FEA predicted (blue) and analytical predicted (green) with the error between the two as a percentage of rated torque in pane 2. Pane 3 is the FFT of both the FEA and analytical cogging torques with the error between the two in pane 4.
6.5.3 Analytical Results

Figure 6.50 shows the effect of static eccentricity on the 10,000 random PMSMs, with the rotor affected 1st and 2nd order sidebands increasing with increasing static eccentricity. While the second order sidebands react significantly, it is difficult to differentiate between stator and rotor affected 2nd order sidebands as the 22nd order harmonic is a 2nd order sideband component common to both and the 20th and 24th.
Figure 6.50  Superposition results for 10,000 randomly generated PMSMs with increasing static eccentricity misalignment. Note the presences of second order sidebands and first order sidebands about the 24th harmonic which, on average, increase with increasing misalignment. Red lines contain 95% of data and blue dashed lines 68%.
6.5.4 Experimental Results

Static eccentricity was experimentally induced by shifting the rotor shaft from one side of the test rig to the other in 0.125 mm increments as outlined in section 5.6.4. As the precise location of the stator in relation to its perfect centre was unknown, this method ensured that the centre position was measured to within 0.06 mm. Figure 6.51 establishes the relationship between the critical orders and static eccentricity and the aligned position can be clearly seen at test 23 where the first order sidebands of the 24\textsuperscript{th} harmonic are a minimum.
Figure 6.51. Experimental static eccentricity testing. Starting at test 1 with the rotor as far to one side of the experimental rig as possible, the rotor was parallel but offset from the stator and incremented 0.125 mm per test until it reach the opposite side. Then central location can be seen at test 23 where the 24th harmonic first order sidebands are a minimum.

Plots of the magnitudes of sideband harmonics in relation to the radial distance of the stator from the ideal position, that is eccentricity, are shown in Figure 6.52. It shows that as static eccentricity increases so too does the magnitude of the 1st order sideband of the 24th harmonic and that the first order sideband harmonics are a reliable indicator of the presence of static eccentricity for a rotor / stator combination whilst the presence of second order sidebands confirms that this is an eccentric misalignment, as these sidebands did not appear when only angular misalignment was present. Noteworthy is the fact that the first
order sidebands of the 24\textsuperscript{th} have a near linear relation to rotor eccentricity for misalignments up to 1.5 mm, after which there is a reduced effect. However, second order sidebands are negligibly affected by small offsets up to 0.8 mm, after which there is a linear relationship between offset and second order sideband of the 24\textsuperscript{th} magnitude.
Figure 6.52 Experimental results for 1st order static eccentricity. Green circles are experimental data, solid red lines contain 95% of analytical data and blue dashed lines 68%.
6.5.5 Discussion of Static Eccentricity Results

The analytical model predicted that both first and second order sidebands of the 24\textsuperscript{th} order harmonic would increase with increasing static eccentricity supporting the work presented in earlier research [110]. These data also correlated well with the experimental data indicating that the static eccentricity experimental testing method appears to be of high quality with a majority of the data falling within the 68\% boundaries of the analytical data. This suggests that the method of inducing the eccentricity to the test rig was effective.

The 24 slot 20 pole PMSM at the focus of this research has the 22\textsuperscript{nd} order being a common 2\textsuperscript{nd} order harmonic to both the rotor and stator affected harmonics, thus it influences both the 2\textsuperscript{nd} order sidebands of the 20\textsuperscript{th} and 24\textsuperscript{th}. However, the source of the 2\textsuperscript{nd} order sidebands can still be determined by referring to the 1\textsuperscript{st} order sidebands, as there is no overlap between rotor and stator affected 1\textsuperscript{st} order sidebands with the motor considered.

6.6 Dynamic Eccentricity Results

This section presents the results for the dynamic eccentricity analysis and testing. Firstly, the effects of any assumptions listed for the analytical method are quantified, then results for the validation of the method are presented, followed by the analytical method results. Finally, these are compared to the experimentally derived data.

6.6.1 Validation of Assumptions

All the assumptions discussed for the static eccentricity configuration also apply to the dynamic situation and these were addressed in section 6.5.1. Furthermore, as dynamic eccentricity results from the rotor spinning about a
non-geometric centre, increasing or decreasing eccentricity results in a torque arm increase or decrease respectively depending on an individual poles position relative to the eccentricity. This is in addition to the timing and skew alterations that occur for both static and dynamic eccentricity.

The analytical method assumes the change in torque arm can be corrected with a torque arm factor. To model the effect, three pole transition cogging torque waveforms were modelled in FEA.

1. Increased moment arm by 2 mm
2. Normal moment arm
3. Decreased moment arm by 2 mm

The raw data is shown in the top pane of Figure 6.53, followed by the corrected cogging torque in pane 2, where the aligned waveform has been factored to both a larger and smaller moment arm. It shows that the eccentric waveforms are within ±1.5% in the time domain of the aligned condition after correction. In the frequency domain, the difference is approximately ±1%. This also confirms that the majority of the difference due to the dynamic eccentricity is from the change in torque arm rather than any MMF differences or fringing effects generated by the eccentric misalignment of the rotor to the stator.
Figure 6.53 Cogging torque of a pole transition over single stator slot with an aligned and ±2 mm of dynamic eccentricity. Top pane shows the FEA raw data with pane 2 showing the moment arm corrected data and pane 3 the percentage error between the eccentric and aligned conditions. Panes 4 and 5 are the frequency domain cogging torques and percentage error to the aligned condition respectively.
6.6.2 Validation of Method

The accuracy of the analytical method was determined by comparing predicted waveforms to those produced from FEA. Figure 6.54 contains both time and frequency domain data and suggests that the analytical method can replicate FEA data in the time domain to within ±3% of rated torque.
Figure 6.54 Validation of analytical method for Dynamic Eccentricity. Top pane shows FEA predicted (blue) and Superposition predicted (green) with the error between the two as a percentage of rated torque in pane 2. Pane 3 is the FFT of both the FEA and Superposition cogging torque with the error between the two in pane 4.

6.6.3 Analytical Results

The superposition data for the 10,000 random PMSMs (Figure 6.55) indicates that increasing dynamic eccentricity increases the first and second order
sidebands of the stator affected harmonic (20th). The same figure also shows second order sidebands of the rotor affected harmonic (24th) increase, however this is due to the 22nd harmonic being a second order sideband to both the 20th and 24th as discussed in section 6.5.5.
Figure 6.55 Analytical results for 10,000 randomly generated PMSMs with increasing dynamic eccentricity misalignment. Note the presences of second order sidebands and first order sidebands about the 20th harmonic which, on average, increase with increasing misalignment. Mean and error bars with ±1 standard deviation.
6.6.4 Experimental Results

Experimental dynamic eccentricity results were obtained, as described in detail in section 5.7.4, by using laser cut acrylic magnet formers with increasing levels of pole eccentricity. The 20 poles were loaded on a laser cut rotor backing plate with a 5 mm larger radius than the production rotor so that eccentric poles did not overhang the edge of the rotor.

Figure 6.56 indicates the relationship between dynamic eccentricity and the critical orders and shows that second order sidebands are present along with first order sidebands about the 20\textsuperscript{th} harmonic, in good agreement with the trends noted for the hybrid FEA / analytical results in the previous section. As with previously presented experimental data, while the relationships between dynamic eccentricity and sidebands of the stator affected harmonics are clearly present, there is more noise reflecting real world testing with some level of mixed misalignment.
Figure 6.56 Experimental results for first order dynamic eccentricity (green circles). Note the presence of second order sidebands which occur when the misalignment is eccentric and first order sidebands of the 20th harmonic which occur for dynamic misalignments. Red lines bound 95% of the analytical data, blue lines 68%.
6.6.5 Discussion of Dynamic Eccentricity Results

The experimental results for dynamic eccentricity correlated well with the analytical data aside from the magnitudes of the experimental 24\textsuperscript{th} order sidebands, however, the causes of the larger experimental data were discussed in section 6.3.5.

As was the case with static eccentricity, the magnitudes of the sidebands does not offer much indication to the degree of misalignment. This is best illustrated by considering the range of 1\textsuperscript{st} order sidebands of the 20\textsuperscript{th} harmonic (from Figure 6.55) where an induced 1 mm eccentricity can have sideband magnitudes of 10 mNm up to 50 mNm. Conversely, if a known sideband magnitude is measured, it is difficult to determine the exact magnitude of the eccentricity as the possible range is significant. For example, if a 1\textsuperscript{st} order sideband of the 20\textsuperscript{th} harmonic is measured at 50 mNm, the possible range of the eccentricity is approximately 1.2 mm to over 2 mm. Therefore, while the presence of 1\textsuperscript{st} and 2\textsuperscript{nd} order sidebands of the stator affect harmonic is an excellent indicator of dynamic eccentricity, the exact magnitude of the sideband is not necessarily an indicator of the magnitude of the misalignment.

6.7 Results Overall Summary

A method capable of predicting cogging torque with a combination of manufacturing errors to within ±5% accuracy was developed and validated. The results from the predictions supported the work completed by others [19, 33, 102, 103, 106-108, 123, 138-143] and specifically that conducted by Gašparin et. al. [18, 144, 145], however, it also considered multiple manufacturing errors in the same simulation.
6 Results

The developed analytical method relied on the FEA created library of pole transition waveforms and the selection of the ‘best fit’ pole transition to suite the required pole spacing for the PMSM being simulated. The timing of the pole transitions were adjusted to simulate the effect of static and dynamic eccentricity with dynamic eccentricity also requiring the change in moment arm to be accounted for. The air gap was adjusted to simulate the effect of static and dynamic angular misalignment.

10,000 randomly generated PMSMs with both pole and slot misplacement were generated and the effect of dynamic and static eccentricity and angular misalignment evaluated independently. The simulation method was orders of magnitude faster than FEA alone.

One downside to the approach implemented was that the relationship between the magnitudes of the relevant sidebands and the magnitudes of the misalignment was unable to be determined. This was due to the large range of sideband magnitudes possible for a particular misalignment. For example, considering a static eccentricity misalignment of 2 mm (Figure 6.50), the magnitudes of the first order sidebands of the 24th harmonic could be anything from 20 mNm to 90 mNm. Also, the sideband magnitude was not a function of the magnitude of the fundamental, as can be seen in Figure 6.57, where the magnitude of the 20th or 24th harmonic for 2.0 mm of static eccentricity is not related to the magnitude of the significant sideband.
Figure 6.57 24th order sideband magnitudes versus 20th order (top pane) and 24th order (lower pane) fundamentals, indicating that there is no relationship between the magnitudes of the fundamental and the magnitudes of the sidebands.
Chapter 7

Implications for Manufacturers

This chapter will briefly analyse and compare the analytical data at the maximum misalignment condition to the aligned condition for each of the four misalignments considered so as to develop generalisations which can be used to construct a fault diagnosis flow chart.

7.1 Misalignment Effects on Critical Orders

The influence of misalignment on the critical orders of the analytical data can be seen in Figure 7.1, where the effect of 0.8° of static angular misalignment can be compared to the aligned condition. These data were generated by averaging the cogging torque in the frequency domain of all 10,000 PMSMs simulated using the
analytical method and subtracting the critical order harmonics of the aligned PMSM from the misaligned, thereby providing an indication of the change in the critical order harmonics as a result of the misalignment investigated. Figure 7.2 to Figure 7.4 show the results for the same analysis on dynamic angular, and static and dynamic eccentricity misalignment respectively.

For the PMSM investigated, 0.8° is close to the maximum possible angular misalignment without the PMs interfering with the stator. While eccentricities greater than 2 mm are possible in axial PMSMs, this was deemed a reasonable upper limit given the production tolerances within the motor. It is apparent that the eccentricity misalignments simulated have a far greater influence on additional cogging torque harmonics than the 0.8° angular misalignments. Additionally, the data indicates that dynamic misalignments, be they angular or eccentric, induce sidebands about the rotor affected \((N_s)\) harmonics while static misalignments induce sidebands about the stator affected \((N_p)\) harmonics. A summary of the data in the following four figures is presented in Table 4.
Figure 7.1 Effect of 0.8° of static angular misalignment on the critical orders of 10,000 randomly generated PMSMs (averaged). Top pane (blue), 0.8° static angular misalignment, middle pane (green), the critical orders of the aligned configuration and, bottom pane (red), misaligned configuration with the aligned critical orders subtracted, leaving only the change due to misalignment.
Figure 7.2 Effect of 0.8° of dynamic angular misalignment on the critical orders of 10,000 randomly generated PMSMs (averaged). Top pane (blue), 0.8° dynamic angular misalignment, middle pane (green), the critical orders of the aligned configuration and, bottom pane (red), misaligned configuration with the aligned critical orders subtracted, leaving only the change due to misalignment.
Figure 7.3 Effect of 2 mm of dynamic eccentricity misalignment on the critical orders of 10,000 randomly generated PMSMs (averaged). Top pane (blue), 2 mm dynamic eccentricity misalignment, middle pane (green), the critical orders of the aligned configuration and, bottom pane (red), misaligned configuration with the aligned critical orders subtracted, leaving only the change due to misalignment.
Figure 7.4 Effect of 2 mm of static eccentricity misalignment on the critical orders of 10,000 randomly generated PMSMs (averaged). Top pane (blue), 2 mm static eccentricity misalignment, middle pane (green), the critical orders of the aligned configuration and, bottom pane (red), misaligned configuration with the aligned critical orders subtracted, leaving only the change due to misalignment.
7 Implications for Manufacturers

7.1.1 Manufacturing Error Effects on Cogging Torque – Summary

The results from sections 6.1 to 6.6 and section 7.1 are summarised in Table 4.

Table 4 Summary of results.

<table>
<thead>
<tr>
<th>Manufacturing Error</th>
<th>Affected Harmonic General Case</th>
<th>Affected Harmonic Research Motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole misplacement</td>
<td>Stator affected harmonics</td>
<td>24</td>
</tr>
<tr>
<td>Slot Misplacement</td>
<td>Rotor affected harmonics</td>
<td>20</td>
</tr>
<tr>
<td>Static Angular Misalignment</td>
<td>1\textsuperscript{st} order sidebands about the N_s harmonic</td>
<td>23\textsuperscript{rd} &amp; 25\textsuperscript{th}</td>
</tr>
<tr>
<td>Dynamic Angular Misalignment</td>
<td>1\textsuperscript{st} order sidebands about the N_p harmonic</td>
<td>19\textsuperscript{th} &amp; 21\textsuperscript{st}</td>
</tr>
<tr>
<td>Static Eccentricity Misalignment</td>
<td>1\textsuperscript{st} and 2\textsuperscript{nd} order sidebands about the N_s harmonic</td>
<td>23\textsuperscript{rd} &amp; 25\textsuperscript{th} and 22\textsuperscript{nd} &amp; 26\textsuperscript{th}</td>
</tr>
<tr>
<td>Dynamic Eccentricity Misalignment</td>
<td>1\textsuperscript{st} and 2\textsuperscript{nd} order sidebands about the N_p harmonic</td>
<td>19\textsuperscript{th} &amp; 21\textsuperscript{st} and 18\textsuperscript{th} &amp; 22\textsuperscript{nd}</td>
</tr>
</tbody>
</table>

7.2 Axial Flux PMSM Cogging Torque Fault Diagnosis Flowchart

A fault diagnosis flow chart based on the analysis conducted in this chapter and results presented in Table 4 is shown in Figure 7.5. It is intended to assist motor manufacturers in identifying axial flux PMSM faults and hence the manufacturing process likely to be responsible, permitting a cost effective means of cogging torque reduction.
Figure 7.5 Fault diagnosis flow chart.
7.3 Reanalysis of the Experimental Production Motor Data

The fault diagnosis flow chart will now be used to reanalyse the 100 experimental PMSMs.

Section 6.1 of the previous chapter presented data to validate the method used for decoupling the stator and rotor contributions along with any assumptions that were made. Additionally the cogging torque for 100 production PMSMs were analysed to identify the sources of manufacturing induced cogging torque. At the time no attempt was made to analyse the source of the harmonics that were not associated with the stator or the rotor, however, the fault diagnosis flow chart presented in section 7.2 can now be applied to the unknown harmonic content and the possible causes to determine. Nearly 30% of the additional cogging torque was categorised as ‘Other’, and results of a more detailed analysis (Figure 7.6 and Figure 7.7) indicates that the positioning of the stator slots most likely followed a first order pattern which in turn induced the 24th order sidebands. Although, as was discussed in section 6.3.5, the stator fault signature is identical to static angular misalignment and therefore this may also contribute to small degree.

After taking the stator manufacturing error and / or static angular misalignment of the 100 PMSMs into consideration, the cogging torque that remains unaccounted has been reduced from 30% in section 6.1 to 5.6% (Figure 7.7).
Figure 7.6 100 PMSM experimental data showing the presence of sidebands, the most prominent being the 1st order sidebands of the stator, indicating that the major misalignment of the test rig was static angular.
Figure 7.7 100 production rotor cogging torque sources as a percentage of the total, from experimental data.

7.4 Analytical Method and FEA Performance Comparison

Ansys Maxwell 16.0 (64 bit version) was the software used to construct the 3D FEA models to simulate the four misalignment conditions and provide the comparative data for the analytical models. Sequential magnetostatic analyses were found to provide the most reliable solutions. The models were run on a PC under Windows 7 64, with 32 GB of RAM and a 8 core processor at 3.2 GHz. Each
model consisted of 171,500 tetrahedra and 1.7% mesh energy error, sequential magnetostatic analyses conducted every 0.5°. Total computation time for each of the four complete PMSM models was 50.4 hours (4.2 minutes per step, average). Thus to model all four misalignments required over 200 hours.

By comparison, the analytical method can be broken down into steps. Firstly, the generation of the lookup table using FEA waveforms, also using Ansys Maxwell, consisted of 26,628 tetrahedra at 0.4% energy error. Magnetostatic simulations were performed for all combinations of air gaps (0.1 mm, 0.7 mm, 1.3 mm, 1.9 mm 2.5 mm) and magnet spacings (16°, 17°, 18°, 19° & 20°) over a 25° arc, sampling every 0.5°. Thus a total of 5 x 5 x 50 = 1250 samples were analysed at an average of 28 seconds per step, or 9.7 hours. The cubic spline interpolation refinement in Matlab took 27 seconds, most of which was saving the file to disk. The superposition of the 10,000 randomly generated PMSMs, with 5 levels of angular misalignment (both static and dynamic) required 0.6 to 0.8 seconds for each PMSM, thus all 50,000 waveforms took 2 hours. The eccentricity superposition model was faster, with each PMSM taking 0.15 seconds for all 5 levels of eccentricity, thus all 10,000 random PMSMs with 5 eccentricities required just over 25 minutes to calculate.

These data are summarised in Table 5.
Table 5  Comparison of method computation time.

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of Models Simulated</th>
<th>Total Simulation Time (Hrs.)</th>
<th>Percentage of FEA Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA</td>
<td>1</td>
<td>50.4 hours</td>
<td>-</td>
</tr>
</tbody>
</table>

The following steps are part of the analytical method. Steps 1 and 2 are performed only once as part of the look up table generation. Steps 3 and 4 are performed as required using the lookup table generated in steps 1 and 2.

<table>
<thead>
<tr>
<th>Step 1.</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA (Lookup table generation)</td>
<td>1250</td>
<td>9.7 hours</td>
<td>19.2%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 2.</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cubic Spline Interpolation</td>
<td>-</td>
<td>0.0075 hours</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 3.</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Superposition (angular misalignment)</td>
<td>50,000</td>
<td>2 hours</td>
<td>4%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 4.</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Superposition (eccentric misalignment)</td>
<td>50,000</td>
<td>0.42 hours</td>
<td>0.8%</td>
</tr>
</tbody>
</table>

The data in Table 5 indicates that the analytical method is significantly faster than the full FEA. The 50.4 hours tabulated is for one FEA model and for this research, five models were required for validation purposes, one for each of the misalignments considered and one with perfect alignment, requiring a total of more than 10 days simulation time. By comparison, once the initial FEA library of waveforms is acquired, the superposition simulation can be completed in seconds (or less) for each solution.
7.5 Fault Diagnosis Summary

The results showed that each of the manufacturing errors considered contributed a unique component to the frequency spectrum offering a potential means of fault identification for manufacturers. This opens the possibility of running fast cogging torque optimisations on alternate designs to select the motor configuration offering the least sensitivity to manufacturing tolerances. Alternately, it may also assist manufacturers with establishing suitable manufacturing tolerance specifications required to ensure motors comply with cogging torque design specifications.
Conclusions

The goal of this research as presented in section 1.7 was to present and validate a method for determining the manufacturing parameters responsible for additional cogging torque in production motors, providing motor manufacturers and designers with a tool to allow cogging torque reduction in a cost effective manner.

There were 5 stages in the development of this work. Firstly, 100 production axial flux PMSMs were experimentally measured and four manufacturing errors that contribute to additional cogging torque identified:

1. Stator causes
2. Rotor causes, which were further decoupled into:
   a. Pole misplacement
   b. Pole strength variation
Secondly, to obtain the experimental data, an existing experimental test rig was analysed and significantly modified to accurately measure cogging torque on motors with out of balance forces.

Thirdly, it was identified that FEA alone was not a suitable means for identifying the manufacturing causes of rotor / stator misalignment as many 3D FEA simulations would be time and resource prohibitive. To overcome this problem, an analytical method capable of simulating cogging torque for a PMSM with multiple sources of manufacturing error was developed. The method was accurate to within ±5% and was several orders of magnitude faster than FEA.

Fourthly, the hybrid FEA / analytical method was then used to generate 10,000 axial flux PMSMs cogging torque waveforms for four possible manufacturing induced misalignment faults. Namely:

1. Static angular misalignment
2. Dynamic angular misalignment
3. Static eccentricity misalignment
4. Dynamic eccentricity misalignment

Fifthly, the analysis enabled the development of a ‘Fault Diagnosis Flow Chart’ to assist motor manufacturers with the identification and rectification of manufacturing processes responsible for additional cogging torque harmonics. Additionally, motor designers can use the developed analytical method to conduct fast PMSM design optimisations for cogging torque to select a design that is the least sensitive to manufacturing tolerances or to assist with establishing acceptable manufacturing tolerances to ensure the PMSM meets the cogging torque design criteria.

For the motors tested, it was found that eliminating stator variation would reduce cogging torque by an average of 30%, perfect pole placement would result in, on average, a 21.5% reduction in cogging torque and eliminating pole
strength variations would achieve a 6.4% average reduction. For the PMSMs experimentally tested, the majority of the remainder of additional cogging torque was found to be slot misplacement associated with the ‘punch and wind’ manufacturing process. Static misalignments were found to induce sidebands about the \( N_s \) harmonics, with angular misalignment responsible for first order and eccentricity responsible for first and second order sidebands. Dynamic misalignments induced sidebands about the \( N_p \) harmonics, with angular responsible for first and eccentricity responsible for first and second order sidebands.

### 8.1 Future Research

The analytical method developed to predict the cogging torque from a library of cogging waveforms incorporated pole and slot placement variation, dynamic and static angular and dynamic and static eccentricity misalignments and was accurate to within ±5%. However, the misalignments were treated individually rather than being able to predict mixed misalignment conditions. Future work could focus on combining the four analytical models into one that is capable simulating mixed misalignment conditions in conjunction with pole and slot placement and pole strength variations.

First order sidebands for the research motor were clearly associated with either the rotor or stator affected harmonics as there was a difference of 4 between the pole and slot count (\( N_p = 20 \) and \( N_s = 24 \)). However, for PMSMs with similar pole / slot combinations, such as \( N_s = N_p \pm 1 \), determining the source of the sidebands would be more difficult as only one side of the first order sidebands would be unaffected. A possible means of clarifying the source of the sidebands would be to consider the sidebands of the second or third harmonics of the \( N_s \) or \( N_p \) orders, as these harmonics would be further separated. For example, a 10 pole 12 slot PMSM has a common middle sideband (the 11\(^{th}\)), however, the
second multiples of $N_s$ and $N_p$ are the 20\textsuperscript{th} and 24\textsuperscript{th} which have greater separation allowing the possible identification of the source of both sidebands.

Additionally, this research was unable to infer the magnitude of the misalignment from the harmonic magnitudes. Future work could investigate if sideband magnitudes can indicate the degree of misalignment if the pole and slot misplacement is quantified through initial or separate testing.

The implemented hybrid FEA / analytical method may be capable of predicting other linear motor parameters for which superposition assumptions apply, such as axial force determination or flux linkage and it is envisaged that the same basic procedure would be suitable. That is, obtain a library of waveforms for the parameter of interest from FEA, post processing to refine the data via cubic spline interpolation, and reassembly of the individual waveforms to generate the complete motor’s signature.

Other research [123] identified that the real and imaginary components of the FFT identified different errors within the PMSM they investigated. It was beyond the scope of this research to evaluate if further information can be obtained by investigating the FFT phase and magnitude components separately, however, further and more accurate decoupling may be possible and would make interesting further study.
References


References


References


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 References


## Appendices

### Appendix A – Test Motor Specifications

**Table 6. Construction data for the PMSM at the focus of the research.**

<table>
<thead>
<tr>
<th>Company, Model</th>
<th>Regal Beloit, Impress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated voltage (V)</td>
<td>240</td>
</tr>
<tr>
<td>Max Power (W)</td>
<td>750</td>
</tr>
<tr>
<td>Rated Torque (Nm)</td>
<td>3</td>
</tr>
<tr>
<td>Speed (RPM)</td>
<td>Variable 1500 - 3000</td>
</tr>
<tr>
<td>Stator Outer Diameter, Inner Diameter, Axial Height (mm)</td>
<td>100, 60, 40</td>
</tr>
<tr>
<td>Number of slots ($N_s$), nominal slot width (mm), depth (mm)</td>
<td>24, 5.2, 24</td>
</tr>
<tr>
<td>Nominal air gap (mm)</td>
<td>1</td>
</tr>
<tr>
<td>Number of poles ($N_p$), nominal thickness (mm), material, skew (deg)</td>
<td>20, 4, Nd$<em>2$Fe$</em>{14}$B, 9.16°</td>
</tr>
<tr>
<td>Least common multiple ($N_p$, $N_s$)</td>
<td>120</td>
</tr>
</tbody>
</table>
Appendix B – Perfect Motor with Misalignment

The results presented in Chapter 6 showed the effects of misalignment on PMSM with random pole and slot placements. This section will show the effects of misalignment on a 20 pole, 24 slot PMSM with perfectly positioned poles and slots. In a manufactured PMSM, some inaccuracy is inevitable therefore this analysis is on a theoretical motor such as is possible in FEA. The same methods as outlined in chapter 5 are applied.

Figure A.0.1 and Figure A.0.2 show that the static misalignment affects the stator affected harmonics while the dynamic affects the rotor harmonics, although the magnitudes of the additional harmonics are small compared to PMSMs with pole and slot placement inaccuracies due to manufacturing tolerances.
Figure A.0.1. Effect of angular misalignment on a perfect PMSM with 20 poles and 24 slots. Note that only the stator affected harmonics are affected by the static misalignment (upper pane) and rotor harmonics by the dynamic misalignment (lower pane).
Figure A.0.2. Effect of eccentricity misalignment on a perfect PMSM with 20 poles and 24 slots. Note that only the stator affected harmonics are affected by the static misalignment (upper pane) and rotor harmonics by the dynamic misalignment (lower pane).
Appendix C - Experimental Test Rig

Several aspects of the experimental test rig proved to be critical, specifically:

1. the ability to accurately measure out of balance forces
2. the ability to easily induce angular and eccentric misalignments

The second of these was presented in detail in sections 4.2.3.2 and 4.4 and will not be discussed further, however, the test rig presented in chapter 4 was a modified version of an existing test rig which was not capable of measuring out of balance forces. The details of the earlier version of the rig will now be presented along with an analysis of its performance.

C.1 Measuring Out of Balance Forces

Design of the experimental setup was critical to the integrity of the data, with particular attention required to ensure out of balance forces did not impair the torque measurement.

The experimental setup presented in chapter 4 was a modified version of the original design. The original design utilised two piezoelectric force transducers mounted vertically as shown in Figure A.0.3. The intention was for these sensors to measure the torque while a pillow block bearing provided axial force reaction to the PM to stator attraction. The advantage of this system is good stiffness and high measurement sensitivity and precision. However, initial testing exposed the transducers sensitivity to cross talk forces. Unfortunately, the published data provided with these transducers did not include any cross talk specifications so this was not easily foreseeable.
In PMSMs utilising fractional pitch, unexpected or additional cogging torque arises from the incomplete cancelation of individual pole cogging torque components in the circumferential direction. At the same time, however, there are also variations in forces in the axial direction between the stator and rotor. If these axial loadings are balanced around the face of the rotor, then there is no influence on the measurement system. However, due to the positioning of the pillow block bearing, any out of balance axial loading also influences the load on the piezoelectric sensors. For example, a larger axial force on the top of the rotor than at the bottom would result in compression of the sensors due to the torque moment arm that is present between the axial reaction force at the bearing, as the force vectors in Figure A.0.3 show. Conversely, larger axial forces at the bottom of the stator produce tensile loads on the transducers and axial loads of the left and right sides will result in shear loading of the transducers. As the location of the out of balance axial forces can never be accurately predicted this system was incapable of providing torque only (circumferential) measurements.
Figure A.0.3. Original experimental setup design with vertically mounted PCB piezoelectric force transducers (circled) and rear mounted bearing to take the thrust load. The rotor side of the setup is unchanged from the final design.

The out of balance forces are critical. Additional components of cogging torque arise due to incomplete cancelation of torque forces, however, axial forces also exist and are constantly varying with rotor position. Inaccuracy could result if these axial forces affect the torque measurement due to sensor cross talk and / or inappropriate setup design.