CHARLES DARWIN UNIVERSITY

Family Name	
Given Names	
Student Number	
Teaching Period	Semester 2, 2016

FINAL EXAMINATION	DURATION
ENG252 – Dynamics	Reading Time: **10** minutes Writing Time: **180** minutes

INSTRUCTIONS TO CANDIDATES

This examination has six questions. Answer **ALL** questions.

The total mark for this examination is **75 marks**.

Note that questions **ARE NOT OF** equal value.

Read all questions carefully, and write your answers CLEARLY in the provided booklet.

EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a RESTRICTED OPEN BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
No additional printed material is permitted	1 x 20 Page Book

THIS EXAMINATION IS PRINTED

DOUBLE-SIDED.

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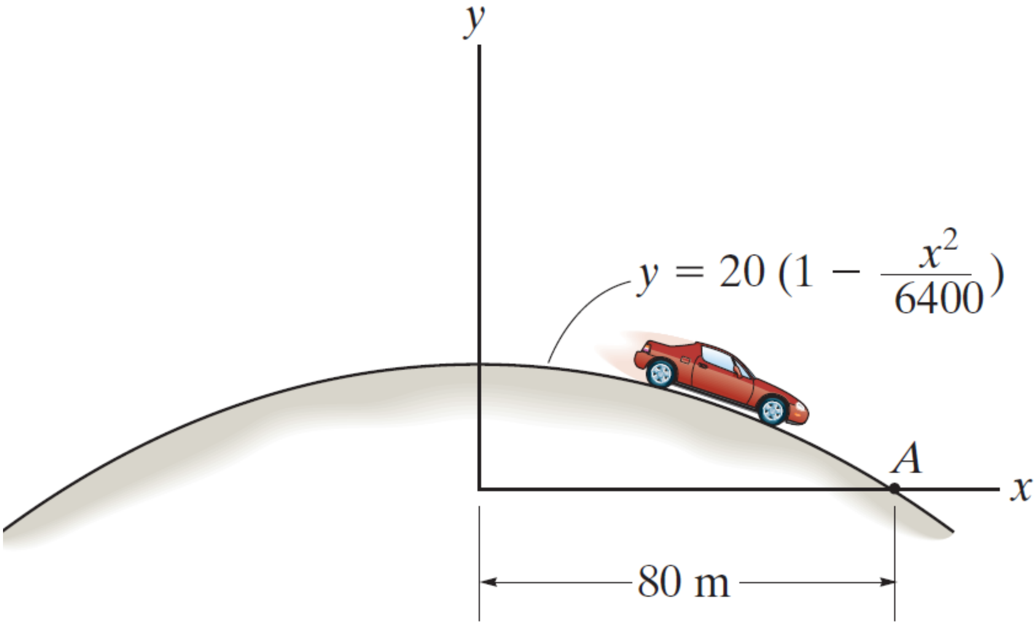
Question 1

The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point *A*, it is traveling at 9 m/s and increasing its speed at 3 m/s^2. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant.

Neglect the size of the car.

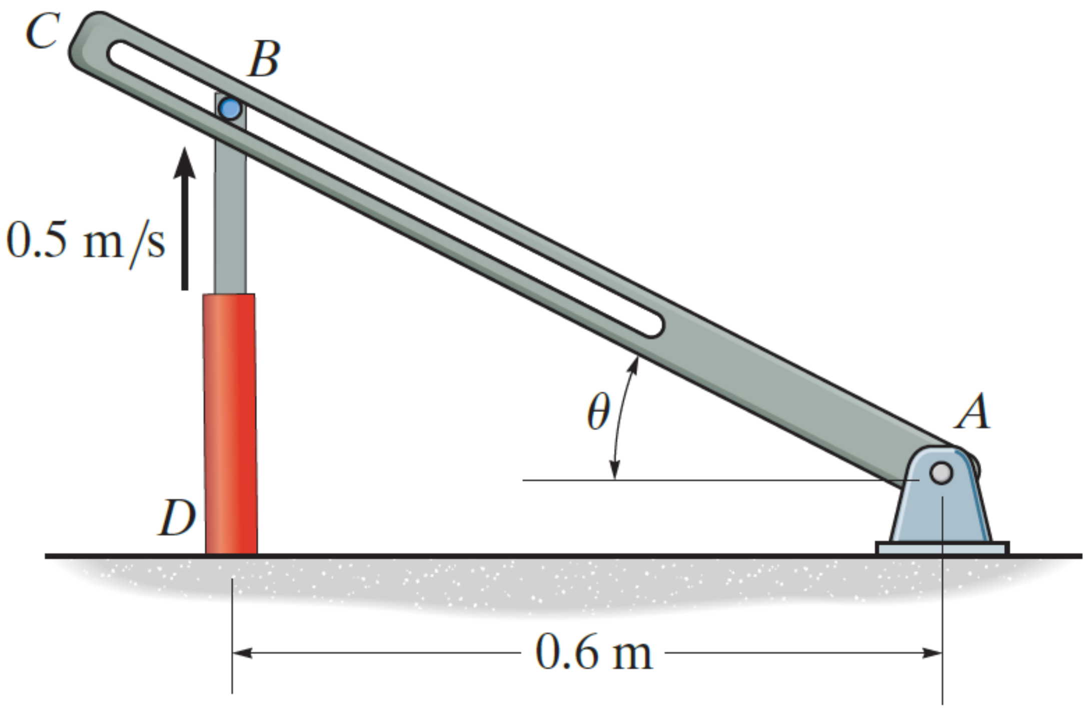
(Marks:10)



Question 2

Peg B mounted on hydraulic cylinder BD slides freely along the slot in link AC. If the hydraulic cylinder extends at a constant rate of 0.5 m/s, determine the angular velocity and angular acceleration of the link at the instant $\theta = 45°$.

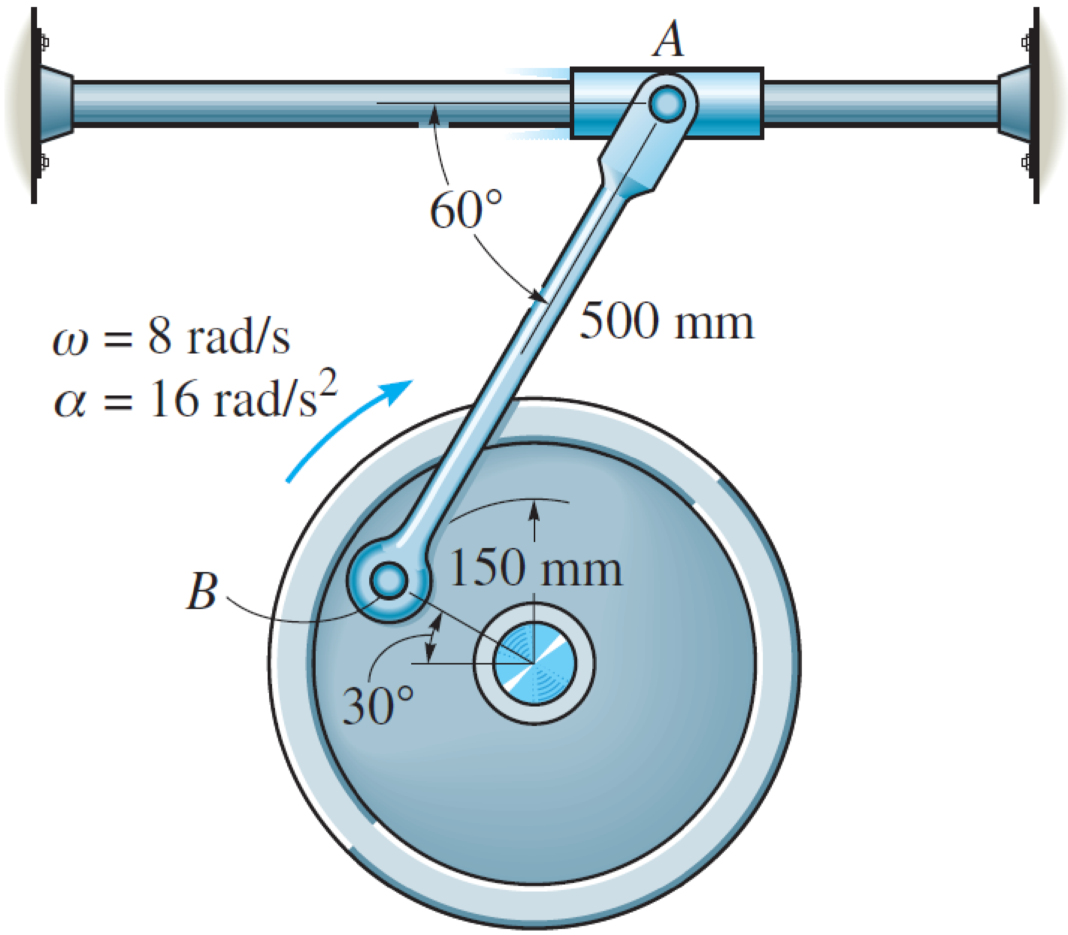
(Marks:10)



Question 3

At a given instant the wheel is rotating with the angular motions shown. Determine the acceleration of the collar at A at this instant.

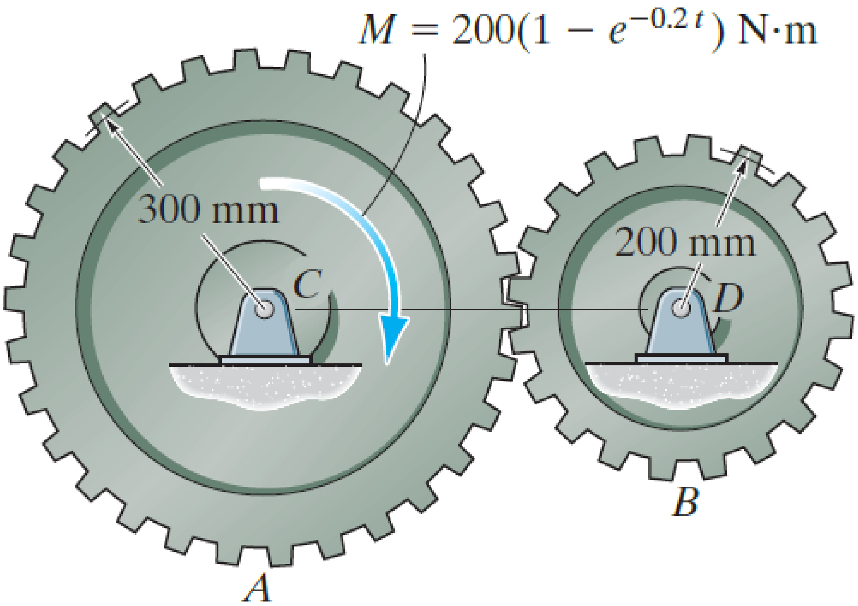
(Marks:10)



Question 4

Gears *A* and *B* have a mass of 50 kg and 15 kg, respectively. Their radii of gyration about their respective centres of mass are k_C = 250 mm and k_D = 150 mm.
If a torque of $M = 200(1\text{-}e^{-0.2t})$ N·m, where *t* is in seconds, is applied to *A*, determine angular velocity of both gears when *t* = 3 s, starting from rest.

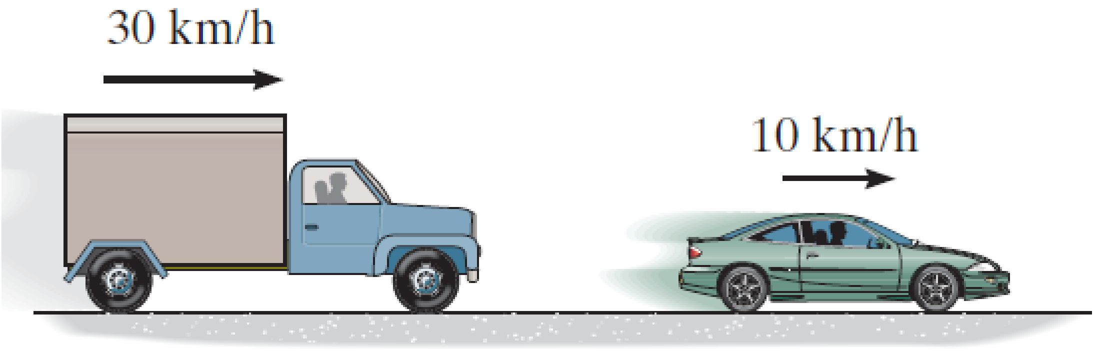
(Marks:15)



Question 5

The 5-Mg truck and 2-Mg car are traveling with the free-rolling velocities shown just before they collide. After the collision, the car moves with a velocity of 15 km/h to the right *relative* to the truck. Determine the coefficient of restitution between the truck and car and the loss of energy due to the collision.

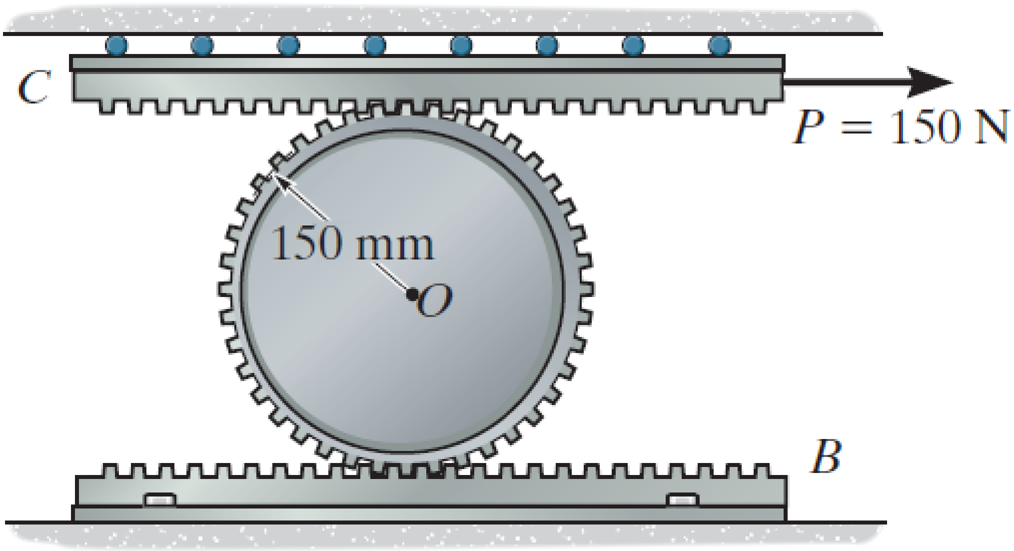
(Marks:15)



Question 6

The 50-kg gear has a radius of gyration of 125 mm about its centre of mass *O*. If gear rack *B* is stationary, while the 25-kg gear rack *C* is subjected to a horizontal force of *P* = 150 N, determine the speed of *C* after the gear's centre *O* has moved to the right a distance of 0.3 m, starting from rest.

(Marks:15)



Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

Variable a	*Constant $a = a_c$*
$a = \dfrac{dv}{dt}$	$v = v_0 + a_c t$
$v = \dfrac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$
$a\,ds = v\,dv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates		*r, θ, z Coordinates*	
$v_x = \dot{x}$	$a_x = \ddot{x}$	$v_r = \dot{r}$	$a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$	$a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$	$a_z = \ddot{z}$	$v_z = \dot{z}$	$a_z = \ddot{z}$

n, t, b Coordinates

$v = \dot{s}$

$a_t = \dot{v} = v\dfrac{dv}{ds}$

$a_n = \dfrac{v^2}{\rho} \quad \rho = \dfrac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$

Relative Motion

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Rigid Body Motion About a Fixed Axis

Variable α	*Constant $\alpha = \alpha_c$*
$\alpha = \dfrac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \dfrac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
$\omega\,d\omega = \alpha\,d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$

Relative General Plane Motion—Translating Axes

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

Relative General Plane Motion—Trans. and Rot. Axis

$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$

$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} \times (\mathbf{a}_{B/A})_{xyz}$

KINETICS

Mass Moment of Inertia $I = \int r^2\,dm$

Parallel-Axis Theorem $I = I_G + md^2$

Radius of Gyration $k = \sqrt{\dfrac{I}{m}}$

Equations of Motion

Particle	$\Sigma\mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$

Principle of Work and Energy

$T_1 + U_{1-2} = T_2$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force	$U_F = \int F\cos\theta\,ds$
Constant force	$U_F = (F_c\cos\theta)\,\Delta s$
Weight	$U_W = -W\,\Delta y$
Spring	$U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$
Couple moment	$U_M = M\,\Delta\theta$

Power and Efficiency

$P = \dfrac{dU}{dt} = \mathbf{F}\cdot\mathbf{v} \quad \epsilon = \dfrac{P_{\text{out}}}{P_{\text{in}}} = \dfrac{U_{\text{out}}}{U_{\text{in}}}$

Conservation of Energy Theorem

$T_1 + V_1 = T_2 + V_2$

Potential Energy

$V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma\int\mathbf{F}\,dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma\int\mathbf{F}\,dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

Coefficient of Restitution $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma\int\mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$
Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma\int\mathbf{M}_G\,dt = (\mathbf{H}_G)_2$ where $H_G = I_G\omega$ $(\mathbf{H}_O)_1 + \Sigma\int\mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = I_O\omega$

Conservation of Angular Momentum

$\Sigma(\text{syst. }\mathbf{H})_1 = \Sigma(\text{syst. }\mathbf{H})_2$

Hibbeler, R.C. Engineering Mechanics Dynamics, 2010

Additional Formulae:

Relative Plane Motion - Translating Axes (B moves along a circular arc with respect to A):

$$\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{B/A}$$

$$\boldsymbol{a}_B = \boldsymbol{a}_A + \boldsymbol{\alpha} \times \boldsymbol{r}_{B/A} - \omega^2 \boldsymbol{r}_{B/A}$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx} \qquad \frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

$$\tan\psi = \frac{r}{dr/d\theta}$$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Integrals:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

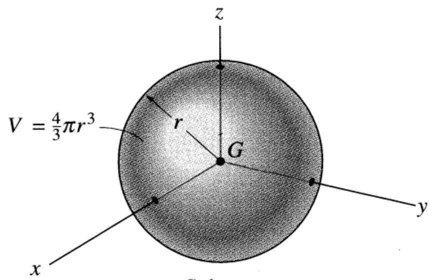
$$\int \frac{dx}{x} = \ln x$$

$$\int \sqrt{a+bx}\,dx = \frac{2}{3b}\sqrt{(a+bx)^3}$$

$$\int x\sqrt{a+bx}\,dx = \frac{2}{15b^2}(3bx-2a)\sqrt{(a+bx)^3}$$

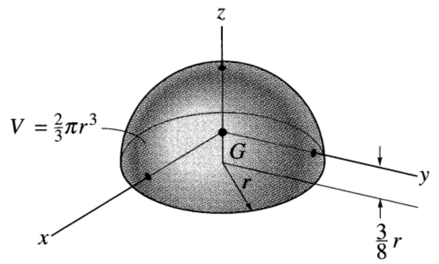
$$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1)$$



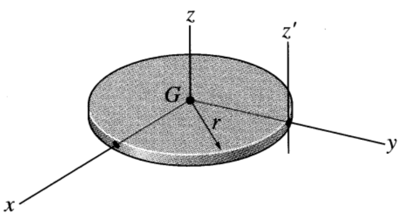
Sphere

$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$



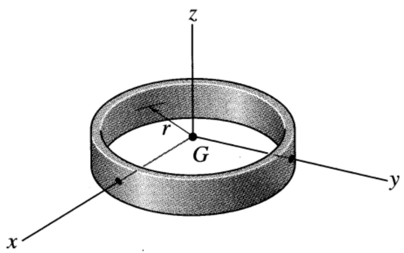
Hemisphere

$I_{xx} = I_{yy} = 0.259mr^2 \quad I_{zz} = \frac{2}{5}mr^2$



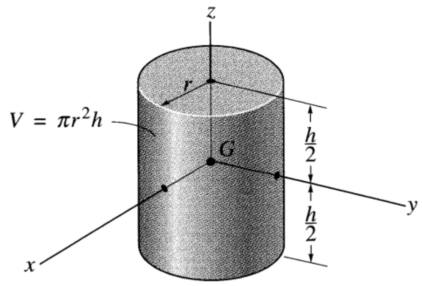
Thin Circular disk

$I_{xx} = I_{yy} = \frac{1}{4} mr^2 \quad I_{zz} = \frac{1}{2}mr^2 \quad I_{z'z'} = \frac{3}{2} mr^2$



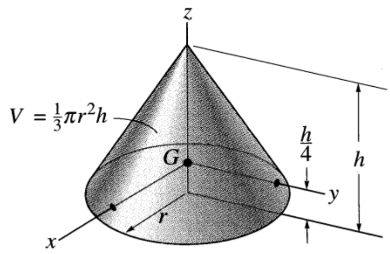
Thin ring

$I_{xx} = I_{yy} = \frac{1}{2} mr^2 \quad I_{zz} = mr^2$



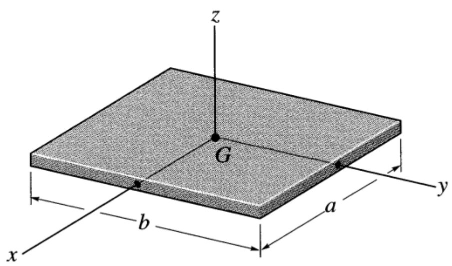
Cylinder

$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2}mr^2$



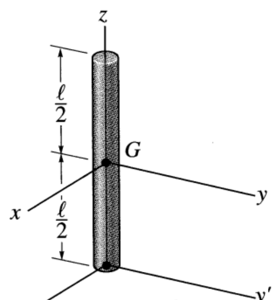
Cone

$I_{xx} = I_{yy} = \frac{3}{80} m(4r^2 + h^2) \quad I_{zz} = \frac{3}{10}mr^2$



Thin plate

$I_{xx} = \frac{1}{12} mb^2 \quad I_{yy} = \frac{1}{12} ma^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$



Slender Rod

$I_{xx} = I_{yy} = \frac{1}{12} m\ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m\ell^2 \quad I_{z'z'} = 0$

Hibbeler, R.C. Engineering Mechanics Dynamics, 2010