CHARLES DARWIN UNIVERSITY

| | |
|---|---|
| **Family Name** | |
| **Given Names** | |
| **Student Number** | |
| **Teaching Period** | Semester 1, 2017 |

| FINAL EXAMINATION | DURATION |
|---|---|
| **ENG421 – Digital Signal Processing** | Reading Time: **10** minutes<br>Writing Time: **120** minutes |

## INSTRUCTIONS TO CANDIDATES

1 Answer all questions

2 This exam constitutes 50% of the total marks for this unit

3 Total number of marks of this exam: 15

Question 1 is worth 3 marks

Question 2 is worth 3 marks

Question 3 is worth 2 marks

Question 4 is worth 2 marks

Question 5 is worth 2 marks

Question 6 is worth 3 marks

## EXAM CONDITIONS

**You may begin writing from the commencement of the examination session.** The reading time indicated above is provided as a guide only.

This is a RESTRICTED OPEN BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

Any hard copy, unannotated English dictionary is permitted

| ADDITIONAL AUTHORISED MATERIALS | EXAMINATION MATERIALS TO BE SUPPLIED |
|---|---|
| No additional printed material is permitted | 1 x 20 Page Book |

**THIS EXAMINATION IS PRINTED DOUBLE-SIDED.**

**THIS PAGE HAS BEEN INTENTIONALLY LEFT BLANK.**

## Question 1 (3 marks)

The frequency response of a linear time-invariant filter is given by the formula:

$$H(\hat{\omega}) = (e^{-3j\pi/2}e^{-j\hat{\omega}} + 1)\cdot(3 + 3\cdot e^{3j\pi/2}e^{-j\hat{\omega}})\cdot(4 + 4\cdot e^{-j\hat{\omega}})$$

**Question 1.1 (1 mark)**
Give the difference equation that gives the relation between the input x[n] and the output y[n].

**Question 1.2 (1 mark)**
Determine the output if the input is:

$$x[n] = \delta[n+2]$$

Sketch the output y[n] for $-4 \le n \le 4$

**Question 1.3 (1 mark)**
If the input is of the form

$$x[n] = Ae^{j\phi}e^{i\hat{\omega}n}$$

for which values of $\hat{\omega}$, with $-\pi \le \hat{\omega} \le \pi$, will $y[n] = 0$ for all n?

## Question 2 (3 marks)

A system is defined by the following z-domain transfer function:

$$H(z) = \frac{(2 + z + z^{-1})\cdot(1 + z^{-2})\cdot(1 - z^{-1})}{(1 + z)}$$

**Question 2.1 (1 mark)**

Determine the difference equation describing this system, with x[n] as input and y[n] as output.

**Question 2.2 (1 mark)**

Determine the frequency domain response of this system and derive two simple formulas (without complex terms and without square roots) for the magnitude versus $\hat{\omega}$ and the phase versus $\hat{\omega}$.

**Question 2.3 (1 mark)**

Determine the output of the system y[n] if the input to the system is $x[n] = 2\cdot\cos(\pi\cdot\frac{n}{4})$.

## Question 3 (2 marks)

Given the feedback filter defined by the difference equation:

$$y[n] = -1.4 \cdot y[n+2] + x[n+2] - 2 \cdot x[n]$$

**Question 3.1 (1 mark)**
Determine the z-transform operator representation H(z) for the system described by the difference equation.

**Question 3.2 (1 mark)**
Determine the poles and zeros of the system. Sketch a pole-zero plot of the system.

## Question 4 (2 marks)

A linear time-invariant system has the impulse response

$$h[n] = u[n] \cdot (-\frac{1}{\sqrt{2}})^n$$

where u[n] denotes the unit step signal.

**Question 4.1 (1 mark)**
Determine the z-transform and frequency domain response of the system described by the impulse response.

**Question 4.2 (1 mark)**
The input to the system is:

$$x[n] = -\cos\left(\frac{\pi(4 \cdot n - 1)}{2}\right) + 3$$

Determine an equation for the output of the system y[n] corresponding to the above input x[n]. Give an equation for y[n] that is valid for all n. (Hint: Use superposition to solve this problem.)

## Question 5 (2 marks)

A 10 point running sum filter with input x[n] and output y[n] is applied to the following two input signals in Question 5.1 and Question 5.2.

### Question 5.1 (1 mark)

Determine the values of y[n] for $-2 \le n \le 14$ with the input signal x[n]:

$$x[n] = \delta[n+1] + \delta[n-3]$$

### Question 5.2 (1 mark)

Determine the response of the filter y[n] to the input signal x[n]:

$x[n] = 10 \cdot \sin(\frac{\pi \cdot n}{4})$ for all n.

## Question 6 (3 marks)

A system has the following transfer function in the z-domain:

$$H(z) = \frac{(1 - j \cdot z^{-1}) \cdot (1 + j \cdot z^{-1}) \cdot (1 - z^{-1})}{(1 - 0.7 \cdot e^{-j\frac{4\pi}{3}} z^{-1}) \cdot (1 - 0.7 \cdot e^{j\frac{4\pi}{3}} z^{-1})}$$

### Question 6.1 (1 mark)

Derive the difference equation describing this system, with x[n] as input and y[n] as output.

### Question 6.2 (1 mark)

Determine all poles and zeros of this system.

### Question 6.3 (1 mark)

If the input of the system is of the form

$$x[n] = A \cdot e^{j\phi} \cdot e^{j\hat{\omega}},$$

for what values of $-\pi \le \hat{\omega} \le \pi$ will y[n] = 0?

## Attached Materials

**SHORT TABLE OF $z$-TRANSFORMS**

| | $x[n]$ | $\Longleftrightarrow$ | $X(z)$ |
|---|---|---|---|
| 1. | $ax_1[n] + bx_2[n]$ | $\Longleftrightarrow$ | $aX_1(z) + bX_2(z)$ |
| 2. | $x[n - n_0]$ | $\Longleftrightarrow$ | $z^{-n_0}X(z)$ |
| 3. | $y[n] = x[n] * h[n]$ | $\Longleftrightarrow$ | $Y(z) = H(z)X(z)$ |
| 4. | $\delta[n]$ | $\Longleftrightarrow$ | $1$ |
| 5. | $\delta[n - n_0]$ | $\Longleftrightarrow$ | $z^{-n_0}$ |
| 6. | $a^n u[n]$ | $\Longleftrightarrow$ | $\dfrac{1}{1 - az^{-1}}$ |

**PROCEDURE FOR INVERSE $z$-TRANSFORMATION ($M < N$)**

1. Factor the denominator polynomial of $H(z)$ and express the pole factors in the form $(1 - p_k z^{-1})$ for $k = 1, 2, \ldots, N$.
2. Make a partial fraction expansion of $H(z)$ into a sum of terms of the form

$$H(z) = \sum_{k=1}^{N} \frac{A_k}{1 - p_k z^{-1}} \quad \text{where} \quad A_k = H(z)(1 - p_k z^{-1})\Big|_{z=p_k}$$

3. Write down the answer as

$$h[n] = \sum_{k=1}^{N} A_k (p_k)^n u[n]$$