CHARLES DARWIN UNIVERSITY

Family Name	
Given Names	
Student Number	
Teaching Period	Semester 1, 2017

FINAL EXAMINATION	DURATION
SMA209 – Mathematics 2A	Reading Time: **10** minutes Writing Time: **180** minutes

INSTRUCTIONS TO CANDIDATES

1 Answer all **six** questions.
2 All questions are of equal value, and parts carry marks as indicated.
3 Read **ALL** questions carefully.
4 Show all working neatly in all parts. Answers without working details will attract little marks.
5 All symbols, unless stated otherwise, have their usual meanings.

EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
No additional printed material is permitted	1 x 20 Page Book 1 x Scrap Paper Formula Sheet/s

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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Question 1

(a) (i) Solve the following ODE by direct integration: **[Marks 3]**

$$y' + xe^{-x^2} = 0.$$

(ii) Is the function $y = (x+c)e^x$ a solution of the following ODE? **[Marks 4]**

$$y' = y + e^x, \ y(0) = \frac{1}{2}.$$

First verify and then determine c.

(b) (i) Write an ordinary differential equation (ODE) for the motion of an object travelling in a straight line, using $y(t)$ as the distance travelled in time t and the condition that distance travelled = velocity x time. **[Marks 4]**

(ii) Solve the above ODE obtained in (i) with the initial condition $y(1) = 1$.

[Marks 4]

(c) Test if the following ODE is exact. If yes then solve it. **[Marks 5]**

$$(x^2 + y^2)dx + 2xydy = 0.$$

Question 2

(a) (i) Solve the following nonlinear first order ODE: **[Marks 6]**

$$2xyy' = y^2 - x^2$$

(ii) Sketch the curve of the solution. **[Marks 4]**

(b) Find the integrating factor for the following ODE and then solve it:

$$2\cosh x \cos y dx = \sinh x \sin y dy.$$ **[Marks 5]**

(c) Find the general solution of the following ODE:

$$xy' = 2y + x^3 e^x.$$ **[Marks 5]**

Question 3

(a) Solve the following initial value problem:

$$y'' + 2y' + 1 = 0. \quad y(0) = 3, y'(0) = 3 \quad .$$ **[Marks 7]**

(b) Solve the following Euler-Cauchy equation for the given initial boundary conditions:

$$x^2 y'' + 3xy' + y = 0, \quad y(1) = 3.6, y'(1) = 0.4.$$ **[Marks 7]**

(c) Find the Wronskian of the following functions and test whether these form basis of solutions:

$$y_1 = e^{-0.4x} \text{ and } y_2 = e^{-2.6x}.$$ **[Marks 6]**

Question 4

(a) Solve the following second order nonhomogeneous ODE:

$$y'' + 5y' + 4y = 10e^{-3x} \quad .$$ **[Marks 7]**

(b) Given that the following functions form the basis of solutions:

e^{-2x} and e^{x}, of the following differential equation:

$$y'' + ay' + b\,y = 0.$$

Find the values of a and b. **[Marks 6]**

(c) Given that $y_1 = x^{-1/2}\cos x$, find y_2 to form the basis of solutions for the following second order ODE.

$$x^2 y'' + xy' + (x^2 - \frac{1}{4})y = 0 \quad .$$ **[Marks 7]**

Question 5

(a) Solve the following initial value problem:

$$\begin{matrix} y_1' = y_2 \\ y_2' = y_1 \end{matrix}, \; y_1(0) = 0, \; y_2(0) = 2.$$ **[Marks 7]**

(b) If f(x) is a periodic function of *x* with period *p*, show that *f(ax)* ($a \neq 0$) is a periodic function of *x* with period *p/a*. **[Marks 5]**

(c) (i) Sketch the following function:

$$f(x) = \begin{cases} x & \text{if } -\pi < x < 0 \\ \pi - x & \text{if } 0 < x < \pi \end{cases}$$ **[Marks 3]**

(ii) Write the Fourier series of $f(x)$ in (i) and determine the Fourier coefficients a_0, a_1 and b_1. **[Marks 5]**

Question 6

(a) What curve is represented by the following vector function:

$r(t) = \cos t\,\mathbf{j} + (2 + 2\sin t)\mathbf{k}$? Sketch the curve. **[Marks 5]**

(b) Find the directional derivative of f at point P in the direction of vector $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, where $f(x, y, z) = x^2 + y^2 + z^2$, and P: (2, 1, 2). **[Marks 5]**

(c) Find the divergence of the following vector function:

$\mathbf{v} = xy^2z^2\mathbf{i} + x^2yz^2\mathbf{j} + x^2y^2z\mathbf{k}$. **[Marks 4]**

(d) Find the line integral $\int_C \mathbf{F}(\mathbf{r}).d\mathbf{r}$ or $\int_a^b \mathbf{F}(\mathbf{r}).\frac{d\mathbf{r}}{dt}dt$ of the following:

$\mathbf{F} = xy\;\mathbf{i} + (y - x)^2\,\mathbf{j}$ on C: $xy = 1$ in the interval $1 \leq x \leq 3$. **[Marks 6]**