Charles Darwin University

Final Examination

Family Name						
Given Name/s						
Student Number						
Teaching Period	Semester 2, 2017					

HIT220 – Algorithms and Complexity	DURATION	
	Reading Time:	10 minutes
	Writing Time:	180 minutes

INSTRUCTIONS TO CANDIDATES

1.1 The examination has 1 section (please adjust details as required if more than one section)

Section A: Suggested Time:180 min	**Short Answer Questions**: Answer ALL 10 questions Marks as indicated by lecturer	**100 Marks**

Section A is to be answered in the Answer Booklet.

Please ensure that your Name and Student Number are written clearly in the space provided at the top of this page.

1.2 Note that questions **ARE NOT** of equal value.
1.3 Read **ALL** questions carefully.
1.4 Do not commence writing until instructed to do so.
1.5 Total Marks for exam: 100 Marks

EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a RESTRICTED OPEN BOOK examination

Any non-programmable calculator is permitted

One A4 sheet of handwritten double-sided notes permitted

Any hard copy, unannotated English dictionary is permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
No additional printed material is permitted	2 x 16 Page Book

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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Section A

Short Answer Questions

Total No of Marks for this section: 100 Marks

This section should be answered in the Answer Booklet provided.

Marks for each question are indicated.

Suggested Time allocation for Section A: 180 mins

QUESTION 1

(a) Calculate the exact number of iterations of the following double loop. Hence, determine the overall complexity of the entire calculation using θ notation.

```
for i := 1 to n-1
    for j := i + 1 to n
        //do something involving θ(n) calculations
    next j
next i
```

(3 marks)

(b) Write out the 5th row of Pascal's Triangle. Explain how one of the entries in out the 5th row of Pascal's Triangle corresponds to the number of iterations of the double loop in part (a) for the case when $n = 5$.

(2 marks)

(c) Calculate the exact number of iterations of the following triple loop. Hence, determine the overall complexity of the entire calculation using Ω notation.

```
for i := 1 to n
    for j := 1 to 2n
        for k := 1 to 3n
            //do something involving Ω(1) calculations
        next k
    next j
next i
```

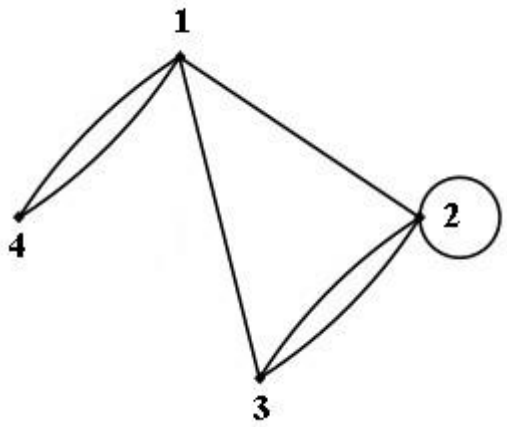
(3 marks)

(d) Is it possible to state the asymptotic complexity of the entire calculation in part (c) using big O notation? Explain.

(2 marks)

QUESTION 2

Consider the following graph G:



(a) Draw an adjacency list representation for G.

(2 marks)

(b) State the adjacency matrix A for G.

(2 marks)

(c) Without calculating A^3, what information does it record about the graph G? State the value of the entry (3,4) of A^3.

(3 marks)

(d) Does G possess an Euler path? Explain.

(3 marks)

QUESTION 3

a) A string is said to be of the form 0^k1^k if it consists of exactly *k* zeros followed by exactly *k* ones.

Write pseudocode for an algorithm which satisfies the following criteria:

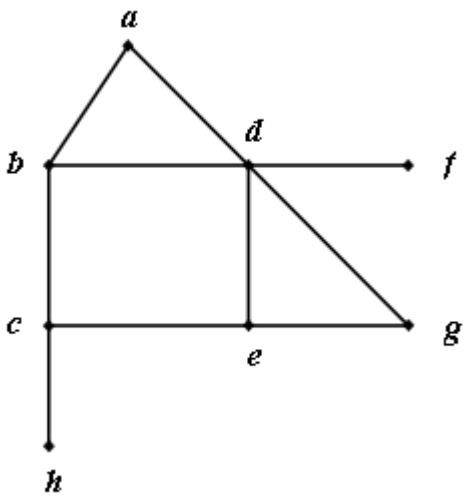
- takes as Input a string (or character array)
- outputs True if the string is of the form 0^k1^k, and otherwise outputs False.

Hint: Use a stack, explicitly or otherwise.

(4 marks)

b) Carry out a **Depth First Search** of the graph below, **starting at vertex b**. Draw a table record the Depth First Index (DFI) as each vertex is visited. Use standard alpha-numeric conventions as appropriate to determine the visitation order. Draw the resultant spanning tree.

(6 marks)



QUESTION 4

a) Write code or pseudocode to recursively compute F_n, the *n*-th Fibonacci number, according to the following definition:

$$F_n = \begin{cases} n & if\ n < 2 \\ F_{n-1} + F_{n-2} & otherwise \end{cases}$$

(2 marks)

b) Comment on the efficiency or otherwise of a recursive algorithm for F_n.

(2 marks)

c) Write an iterative algorithm that uses an array to calculate F_n. State and briefly explain its order of complexity.

(3 marks)

d) A closed form formula for F_n is given by the formula:

$$F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

Comment on the implications of this closed form formula with respect to the algorithmic complexity of computing F_n for large *n*.

(3 marks)

QUESTION 5

a) Draw the BST constructed from inserting the keys, 13, 23, 18, 3, 8, 20, 16, 19, 21, 27, in that order, into an initially empty tree.

(3 marks)

b) List the visitation sequence for a preorder and a postorder traversal of the BST in Part a).

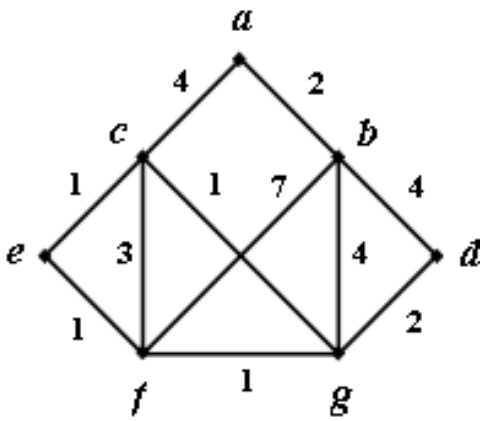
(4 marks)

c) Delete the node with key 23 and draw the resultant tree. State the method you have used to perform the delete operation. Name a second method that can be used for deletion of a key.

(3 marks)

QUESTION 6

Consider the following graph *G*:



a) Find a spanning tree of *G* using **Dijkstra's Distance Algorithm**, beginning at vertex a.

Copy the table below into your exam booklet and use it to record appropriate labels at each stage of the algorithm. Draw the resultant tree in your exam booklet. Use standard alpha-numeric conventions, as appropriate, to determine the visitation order.

Order	**L(*a*)**	**L(*b*)**	**L(*c*)**	**L(*d*)**	**L(*e*)**	**L(*f*)**	**L(*g*)**

(6 marks)

b) What information does the resultant spanning tree record? State an example found by the algorithm to assist your explanation.

(2 marks)

c) Suppose that instead of the information found by Dijkstra's Distance Algorithm, you seek a minimum cost tour of the vertices of an arbitrary graph G that begins and finishes at the same vertex. i.e. a minimum weighted cycle that visits all of the vertices of an input graph G. Would you expect to find an efficient algorithm for this problem? Why or why not?

(2 marks)

QUESTION 7

a) Construct a max heap using the bottom up (Floyd) algorithm on the keys 9,12,6,14,11,5, inserted in that order. As each key is inserted and/or swapped, draw the binary tree and show the current state of the array.

(4 marks)

b) State, along with brief explanation, two algorithms or applications where a min heap is an appropriate data structure for the job.

(3 marks)

c) Write a recursive algorithm for computing the number of leaves in a binary tree.

(3 marks)

QUESTION 8

a) Write pseudocode for an algorithm which uses an ordered array to perform global balancing of a BST. What are the advantages and disadvantages of this method?

(4 marks)

b) Demonstrate the Insertion Sort Algorithm using an array for an unsorted list 10,5,11,7, 8. At each stage of the algorithm, draw the array and identify which the entries are sorted or unsorted.

(3 marks)

c) State and briefly explain the best and worst case time complexity for Insertion Sort.

(3 marks)

QUESTION 9

a) Binary Search is and algorithm which can be classified as "divide and conquer" algorithm. State and explain the best and worst case time complexity of Binary Search. Name an example of another "divide and conquer" algorithm and state its worst case complexity.

(4 marks)

b) Name three sorting algorithms which are "in place" algorithms for sorting an unordered array. i.e. The algorithms require little additional storage. State the worst case complexity of each algorithm.

(3 marks)

c) "Nearest Neighbour" is an example of an heuristic algorithm for the Travelling Salesman Problem. Explain the idea of the algorithm. Does it give an exact answer? Is it a greedy algorithm?

(3 marks)

QUESTION 10

a) How can it be shown that a decision problem is in the complexity class P? Give an example of a decision problem in P and demonstrate its membership in that class.

(3 marks)

b) How can it be shown that a decision problem is in the complexity class NP? Give two examples of decisions problem in this class along with brief explanations to demonstrate membership in NP.

(3 marks)

c) Explain the significance of finding a polynomial time algorithm for a problem in the class NPC. What is Cook's Theorem about?

(4 marks)