Charles Darwin University

Final Examination

Family Name						
Given Name/s						
Student Number						
Teaching Period	Semester 2, 2017					

HIT400 – Discrete Structures	DURATION	
	Reading Time:	10 minutes
	Writing Time:	120 minutes

INSTRUCTIONS TO CANDIDATES

1.1 The examination has 1 section (please adjust details as required if more than one section)

Section A: Suggested Time:120 min	**Short Answer Questions**: Answer ALL 10 questions Marks as indicated by lecturer	**100 Marks**

Section A is to be answered in the Answer Booklet.

Please ensure that your Name and Student Number are written clearly in the space provided at the top of this page.

1.2 Note that questions **ARE NOT** of equal value.
1.3 Read **ALL** questions carefully.
1.4 Do not commence writing until instructed to do so.
1.5 Total Marks for exam: 100 Marks

EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a RESTRICTED OPEN BOOK examination

Any non-programmable calculator is permitted

One A4 sheet of handwritten double-sided notes permitted

Any hard copy, unannotated English dictionary is permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
No additional printed material is permitted	2 x 16 Page Book 1 x Scrap Paper

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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Section A

Short Answer Questions

Total No of Marks for this section: 100 Marks

This section should be answered in the Answer Booklet provided.

Marks for each question are indicated.

Suggested Time allocation for Section A:120 mins

Question 1

Consider the partial order relation "divides" on the set of positive factors of 35.

a) Draw the directed graph for the relation and explain how it demonstrates the properties of an order relation.
b) Name the three properties that an equivalence relation must satisfy and state one example of an equivalence relation on the set of natural numbers.

(Marks: 10)

Question 2

Prove by induction that the following statement is true for all natural numbers n:

$$1 + 2 + 3 + \cdots + \mathrm{n} = \frac{n(n+1)}{2}$$

(Marks: 10)

Question 3

Let R be the relation on $\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}$ defined by the following matrix

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} T & F & F & T \\ F & T & F & F \\ F & F & T & F \\ T & F & F & T \end{bmatrix} \end{array}$$

a) Draw the graphical representation of R.
b) State, giving reason, whether R is reflexive, symmetric or transitive. (Marks: 10)

Question 4

For each of the following functions, determine whether the function is onto and whether it is one-to-one:

a) $f: \mathbb{R} \longrightarrow \mathbb{R}, f(x) = 9x + 12$

Let X= {finite no-empty strings of bits} and Y={0,1,2,3,...}

b) $g: X \longrightarrow Y, g(s) =$ number of ones in s

(Marks: 10)

Question 5

Show that the union of any two countable sets is a countable set.

(Marks: 10)

Question 6

Let G_1 and G_2 be isomorphic graphs. Prove that if G_1 is connected, then G_2 is connected.

(Marks: 10)

Question 7

Design a DFA that accepts an input string with a multiple of 3 a's in an input with alphabet of {a,b,c}.

Express this as a DFA that is in terms of $\{Q, \sum, \delta, q_0, F\}$.

(Marks: 10)

Question 8

a) Suppose you offer a choice of coffee, tea or chai, with full milk reduced milk or no milk and either with our without sugar. Express all the possible drinks you could have as a Cartesian product of sets. Show its cardinality.

b) Let n be a positive integer. How many triples of integers (I,j,k) are there with $1 \leq i \leq j \leq k \leq n$?

(Marks: 10)

Question 9

Short paragraph answer.

Recently Boon Boffin proposed P $\neq NP$. If it were true, what would be the consequence of the statement?

(Marks: 10)

Question 10

Solve the following recurrence relations. i.e. express them in non-recursive form.

A statement of O notation bounds is sufficient.

$$\mathbf{T(n)} = \mathbf{T}\left(\frac{\mathbf{n}}{\mathbf{4}}\right) + \boldsymbol{n}, \text{ T(1)=1}$$

(Marks: 10)