# Charles Darwin University

Final Examination

| Family Name | |
|---|---|
| Given Name/s | |
| Student Number | |
| Teaching Period | Semester 2, 2017 |

| SMA102 – Mathematics 1B | DURATION | |
|---|---|---|
| | Reading Time: | **10** minutes |
| | Writing Time: | **180** minutes |

## INSTRUCTIONS TO CANDIDATES

1.1 This paper contains six questions. Answer all **six (6)** questions.
1.2 All questions are of equal value, and parts carry marks as indicated.
1.3 All symbols, unless stated otherwise, have their usual meanings.
1.4 Read <u>ALL</u> questions carefully.
1.5 Answers that do not show detailed working will attract little marks.

## EXAM CONDITIONS

<u>You may begin writing from the commencement of the examination session.</u> The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

| ADDITIONAL AUTHORISED MATERIALS | EXAMINATION MATERIALS TO BE SUPPLIED |
|---|---|
| No additional printed material is permitted | 1 x 20 Page Book<br>1 x Scrap Paper<br>Formula Sheet/s |

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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**Answered ALL questions in the Answer Booklet provided.**

Marks for each question are indicated.

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**Question 1**

(a) Using integration by parts, evaluate the following indefinite integrals:

(i) $\int x^2 e^{2x} dx$ (Mark : 7)

(ii) $\int \sqrt{x} \ln(x) dx$ (Mark : 3)

(b) Using partial fractions, evaluate the following integral:

$$\int \frac{2x+4}{x^3 - 2x^2} \, dx$$ (Mark : 7)

(c) Given that $\ln a = 2$, and $\ln c = 5$, evaluate the following integral:

$$\int_1^{ac} \frac{1}{t} \, dt$$ (Mark : 3)

**Question 2**

(a) Evaluate the following integrals:

(i) $\int \tan(4x)\sec^4(4x)dx$

(ii) $\int (\ln x)^2 dx$ (Mark : 7)

(b) (i) Sketch the region enclosed between the curve $y = \sqrt{x}$ , $x = 1$, $x = 4$ and the x-axis. (Mark : 3)

(ii) Use the cylindrical shells method to determine the volume of the solid generated when the region in (i) is rotated about the $y$ -axis. (Mark : 4)

(c) Find the exact arc length of the curve over the stated interval:

$y = 3x^{3/2} - 1$ from x = 0 to x=1. (Mark : 6)

**Question 3**

(a) First sketch the two curves $y_1 = \sin x$ and $y_2 = \cos x$ from $x = 0$ to $x = 2\pi$ and then find the area enclosed between them. (Marks : 8)

(b) Apply the ratio test for absolute convergence to show that the power series representation of the Bessel function of the first kind of order 1, $J_1(x)$, converges for all $x$.

$$J_1(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2^{2k+1}(k!)(k+1)!}$$ (Marks : 5)

(c) Determine whether the series converges, and if so find its sum.

$$\sum_{k=1}^{\infty} \left( \frac{1}{5^k} - \frac{1}{k(k+1)} \right)$$ (Marks : 7)

**Question 4**

(a) Find the Taylor series of $f(x) = \frac{1}{x}$ at $x_0 = -1$, and write it in sigma notation.

(Marks : 7)

(b) Evaluate the following improper integral:

$$\int_3^{+\infty} \frac{2}{x^2 - 1} dx$$ (Marks : 6)

(c) Sketch the following functions $f(x)$ and $g(x)$ first. Then find the volume of the solid generated when the region between the graphs of the two functions $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval [0, 2] is revolved about the x-axis.

(Marks : 7)

**Question 5**

(a) Find the standard matrix for the linear transformation defined by the following equations:

(i)
$$\begin{aligned} w_1 &= 7x_1 + 2x_2 - 8x_3 \\ w_2 &= \quad\; - x_2 + 5x_3 \\ w_3 &= 4x_1 + 7x_2 - x_3 \end{aligned}$$
(Marks : 4)

(iii) For which values of k are **u** = (k, k, 1) and **v** = (k, 5, 6) orthogonal ? (Marks : 4)

(b) Let $\mathbf{u}_1 = (-1, 3, 2, 0)$, $\mathbf{u}_2 = (2, 0, 4, -1)$, $\mathbf{u}_3 = (7, 1, 1, 4)$, and $\mathbf{u}_4 = (6, 3, 1, 2)$.

Find scalars $c_1$, $c_2$, $c_3$, and $c_4$ such that $c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3 + c_4\mathbf{u}_4 = (0, 5, 6, -3)$

(Marks : 8)

(c) Show that the vectors $\mathbf{v}_1 = \left(-\frac{3}{5}, \frac{4}{5}, 0\right)$, $\mathbf{v}_2 = \left(\frac{4}{5}, \frac{3}{5}, 0\right)$, $\mathbf{v}_3 = (0,0,1)$ form an orthonormal basis for $R^3$ with the Euclidean inner product. (Marks : 4)

**Question 6**

(a) Find a basis for the null space of $A$ given by:

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$$ (Marks : 6)

(b) Use the Gram-Schmidt process to transform the basis $\{\mathbf{u}_1,\ \mathbf{u}_1,\ \mathbf{u}_1\}$ in $R^3$ into an orthonormal basis, where $\mathbf{u}_1 = (1, 1, 1)$, $\mathbf{u}_2 = (-1, 1, 0)$ and $\mathbf{u}_3 = (1, 2, 1)$. (Marks : 7)

(c) Find a matrix $P$ that can diagonalise $A$ given by:

$$A = \begin{bmatrix} 6 & 2\sqrt{3} \\ 2\sqrt{3} & 7 \end{bmatrix}$$ (Marks : 7)

Calculate $P^{-1}AP$.