Charles Darwin University

Final Examination

Family Name						
Given Name/s						
Student Number						
Teaching Period	Semester 2, 2018					

ECM262 – Teaching the Curriculum: Senior Secondary Mathematics	DURATION	
	Reading Time:	10 minutes
	Writing Time:	120 minutes

INSTRUCTIONS TO CANDIDATES

There are three sections to this exam. You need to answer all three.

- **Section A** is of 37 marks, this section deals with your basic and senior mathematics content knowledge.
- **Section B** is of 14 Marks, it deals with writing model solutions. You need to answer any two out of the four questions in this section.
- **Section C** is of 24 marks, this section is concerned with your pedagogical content knowledge and assessment of students' work.

Please note that under the authorised materials, Lecture Text books include any Senior Maths Text Books.

EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a RESTRICTED OPEN BOOK examination

Any calculator is permitted

One A4 sheet of handwritten double-sided notes permitted

Any hard copy, unannotated English dictionary is permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
Lecture Notes (Unannotated) Lecture Textbook/s (Unannotated) No additional printed material is permitted	1 x 8 Page Book 1 x Scrap Paper

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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Section B
Short Answer Questions
(Writing model solutions)

Marks for each question are indicated.
Suggested Time allocation for Section B: 25 minutes
Total No of Marks for this section: 14

Write model solutions for **any two** of the following four questions **in this section**, assuming these be given to the students to emphasize the importance of communicating mathematically when presenting their solutions

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Question 1

Find $\frac{dy}{dx}$ for each of the following functions. There is no need to simplify your answers.

(a) $y = (x^3 - \frac{3}{x})^6$

Formula

Step 1

Step 2

Answer

(3 marks)

(b) $y = \frac{e^{-x}}{1+\ln(x)}$

Formula

Step 1

Step 2

Answer

(4 marks)

Question 2

Consider D = $\begin{bmatrix} 3 & k & k^2 \\ -3 & 0 & k \\ 0 & 1+k & k^2 \end{bmatrix}$

(a) Evaluate the determinant of D

Formula

Step 1

Step 2

Answer

(4 marks)

(b) Hence determine the value(s) of *k* for which the matrix *D* has no inverse.

Explanation:

When does a matrix not have an inverse?

Step 1

Step 2

Answer

(3 marks)

Question 3

(a) Find the minimum value of $y = \frac{e^x}{sinx} \quad 0 \leq x \leq \pi$ giving a simplified answer in exact form.

Explanation:

- *Finding f'(x)*
- *Equating it to zero and solving for x*
- *Drawing the sign diagram*
- *Finding the min value*

(4 marks)

(b) If $f(t) = 5\sin(3t - 4)$ show that $f''(t) = -9f(t)$.

Explanation:

Finding f'(x)

Finding f"(x)

Simplifying

(c) Find the second derivative of the following functions and summarise your results in the table below:

$f(t)$	$5\sin(3t-4)$	$4sin(2t+5)$	$-\sin(t-3)$	$Asin(\omega t+k)$
$f''(t)$	$-9f(t)$			

Finding f'(x) for f9x) = $4sin(2t+5)$

Finding f'(x) for f9x) = $-sin(t-3)$

Generalizing to $Asin(\omega t+k)$

(3 marks)

Question 4

Consider the 'curve' $xy^2 + 2y - 4 = 0$ where *x* and *y* are real numbers.

(a) Show that $\frac{dy}{dx} = \frac{-y^2}{2(xy+1)}$

Formula *Derivative of implicit functions*
Step 1
Step 2
Answer

(3 marks)

(b) Show algebraically that there are 2 points on the curve where $x = 2$. State the coordinates of each point, and find the slope of the curve at each point.

What happens *at x=2 Substitution in the equation*
Solving *the equation*
Writing *coordinates of points*
Connection *of dy/dx and slope*
Finding *the slopes*

(4 marks)

Section C
Create a response Questions
(Pedagogical knowledge and assessment of students' work)

Marks for each question are indicated.
Suggested Time allocation for Section C: 45 minutes
Total No of Marks for this section: 24

This section is about answering students' queries, assessing their work and providing suitable feedback to them.

Question 1
You have finished teaching a specific unit and are about to begin teaching the topic of **Functions**. How would you initiate, engage and motivate the study of this topic in your class?

(4 marks)

Question 2

a) A student asks you the meaning of Sine, Cosine (in Trigonometry), what is the basic idea? Where they come from? How would you respond?

b) Another student asks you why-we need to memorize 'Times tables' in the age of calculators, mobile phones and other hand held electronic devices, when a push of a few buttons can give us the answer? How would you respond?

(3+3 marks)

Question 3

A student was given the task of solving the following equation

$$x^2 - 8x + 15 = 3$$

The student proceeded with

$$x^2 - 8x + 15 = (x - 3)(x - 5) = 3$$

Thus

$$(x - 3)(x - 5) = 3 = 3 \times 1$$

Thus either

$$(x - 3) = 3 \text{ and } (x - 5) = 1$$

And hence $x = 6$

or $(x - 3) = 1$ *and* $(x - 5) = 3$

Which means $x = 4$ *or* 8

Thus the solutions are $x = 4, 6 \text{ or } 8$

He verifies these :

checks for x = 4 finds $4^2 - 8 \times 4 + 15 = 16 - 8 \times 4 + 15 = 16 - 32 + 15 = -1$ *, thus 4 doesn't work .*

checks for x = 6 finds $6^2 - 8 \times 6 + 15 = 36 - 8 \times 4 + 15 = 36 - 48 + 15 = 3$ *, thus 6 does work .*

checks for x = 8 finds $8^2 - 8 \times 8 + 15 = 64 - 8 \times 8 + 15 = 64 - 64 + 15 = 15$*, thus 15 doesn't work .*

Claims the only solution is $x = 6$

*What is wrong with the solution and <u>how would you **explain the fallacy**</u>? (Note the question is not to solve the question correctly, but to find, where the error in the student's thinking and reasoning is.)*

(5 marks)

Question 4

Consider the following "proof" for 1 = -1 by a student:

"Proof"

1 = -1

Step 1

Since

$$\frac{-1}{1} = \frac{1}{-1}$$

Step 2

Taking the square root of both sides

$$\sqrt{\frac{-1}{1}} = \sqrt{\frac{1}{-1}}$$

Step 3

Then

$$\frac{\sqrt{-1}}{\sqrt{1}} = \frac{\sqrt{1}}{\sqrt{-1}}$$

Step 4

Therefore $\frac{i}{1} = \frac{1}{i}$

Step 5

Multiply both sides by i to obtain

$$\frac{i^2}{1} = \frac{i}{i}$$

Step 6

Simplify each fraction to obtain

$$i^2 = 1$$

But then we know $i^2 = -1$

Therefore -1 = 1

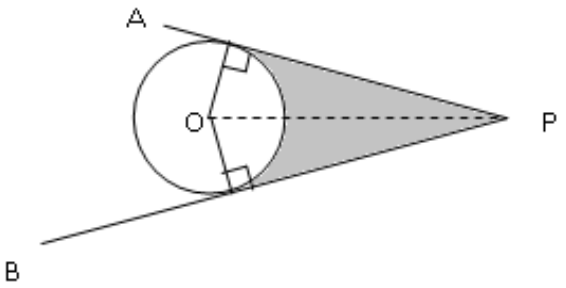
(The complex number i = $\sqrt{-1}$ **)**

Clearly this is not correct. **Identify the first incorrect step.** How *would you explain this to the concerned student* with a focus on <u>what conceptual misunderstanding</u> the student might be having?

(4 marks)

Question 5

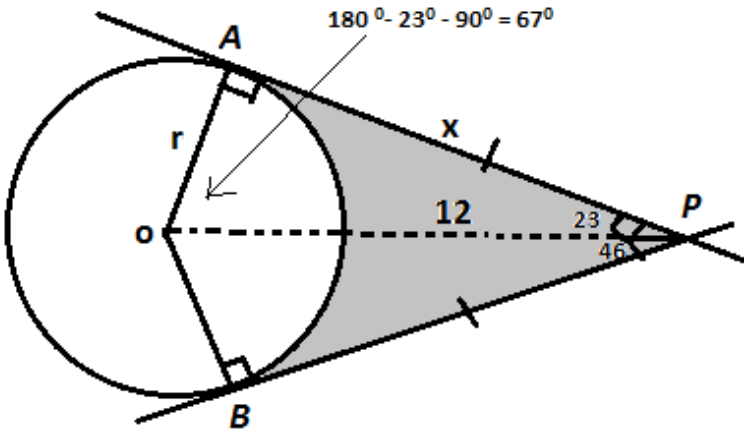
In the following diagram PA and PB are tangents to the circle, OP = 12 and $\angle$ APB = 46^0
Find the shaded Area



Following is the solution provided by a student. You need to assess the solution (mark the steps with a tick if correct, cross the steps that are wrong and write what went wrong and how to improve it next time, finally give a mark) .

(5 marks)

Solution:



Solution:

$\sin 23 = \frac{r}{12}$

$r = 12 \sin 23$

$\doteq 4.69$ cm

Area of Sector $= \frac{\text{Angle}}{360^\circ} \times \pi \times r^2$

$\therefore$ Area of Sector $= \frac{67}{360} \times \pi \times 4.69$

$\doteq 12.9$ cm

$12^2 = x^2 + r^2$

$x^2 = 12^2 - r^2$

$x = \sqrt{12^2 - 4.69^2}$

$\doteq 11.04$ cm

$\therefore$ Area of $\Delta = \frac{1}{2} \times 11 \times 12 \sin 23$

$\doteq 25.9$ cm^2

or Area of $\Delta = \frac{1}{2} \times 4.69 \times 12 \times \sin 67$

$\doteq 25.9$ cm

$\therefore$ Shaded Area $=$ A of Sector + A of Δ

$= -12.9 + 25.9$

$= 25.9 - 12.9$

$= 13$ cm

(It was given as a 4 marks question to the student)

Extra Space if needed