Charles Darwin University

Final Examination

| Family Name | | | | | | |
|---|---|---|---|---|---|---|
| Given Name/s | | | | | | |
| Student Number | | | | | | |
| Teaching Period | Semester 2, 2018 | | | | | |

| ENG252 – Dynamics | DURATION | |
|---|---|---|
| | Reading Time: | 10 minutes |
| | Writing Time: | 180 minutes |

## INSTRUCTIONS TO CANDIDATES

1. Read all questions carefully and write your answers CLEARLY in the provided booklet.
2. This examination has six questions. Answer all questions.
3. Total marks available on this test are 100.
4. Note that questions ARE NOT OF equal value.

## EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

| ADDITIONAL AUTHORISED MATERIALS | EXAMINATION MATERIALS TO BE SUPPLIED |
|---|---|
| No additional printed material is permitted | 1 x 20 Page Book |

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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## Question 1

The 3 kg block *A* is released from rest in the 60° position shown (Figure 1) and subsequently strikes the 1 kg cart *B*. If the coefficient of restitution for the collision is *e* = 0.7, determine the maximum displacement *s* of the cart *B* beyond point *C*. Neglect friction.

(Marks: 20)

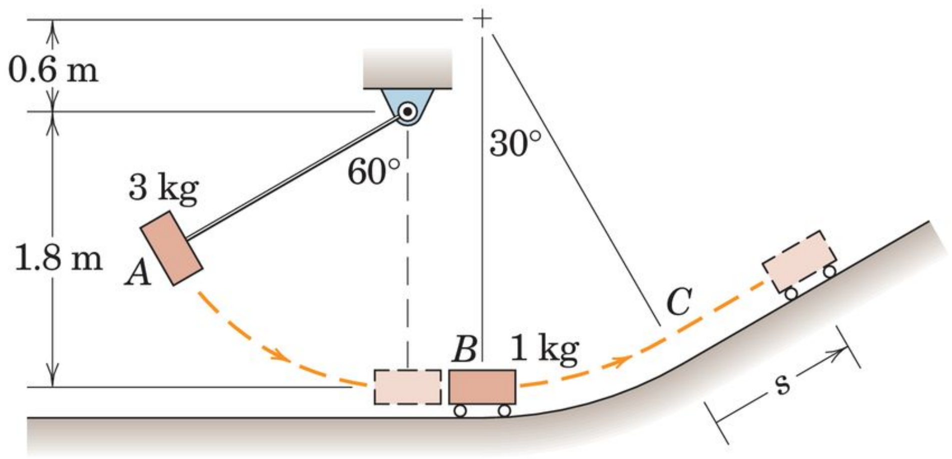



Figure 1*

**Meriam J.L., Kraige L.G.: Engineering Mechanics, Volume 2 Dynamics, Eighth Edition, John Wiley and Sons, 2015*

## Question 2

If the cord is subjected to a constant force of $F = 300$ N and the 15-kg smooth collar starts from rest at *A*, determine the velocity of the collar when it reaches point *B*. Neglect the size of the pulley.

(Marks: 10)

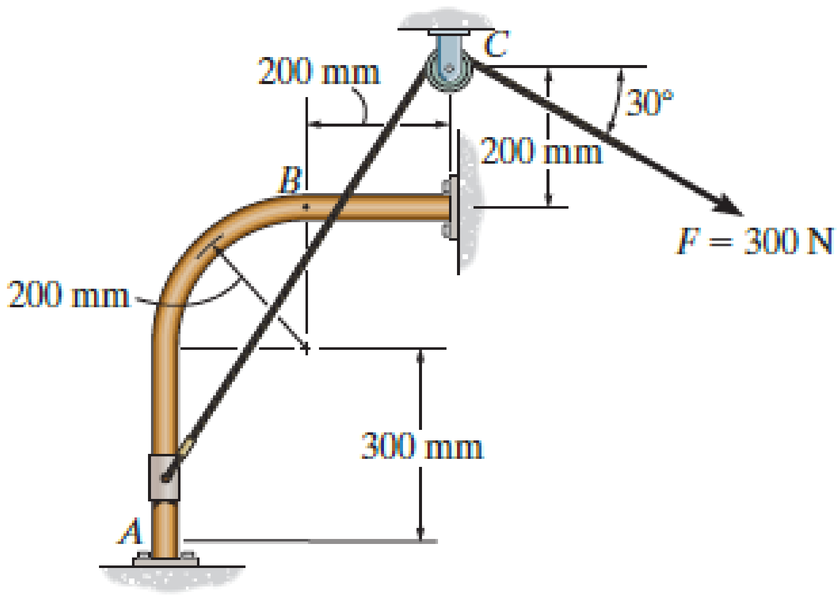



Figure 2**

***Hibbeler R.C.: Engineering Mechanics, Dynamics, Fourteenth Edition, Pearson Prentice Hall, 2016*

## Question 3

At a given instant, the slider block *A* has the velocity and deceleration shown at Figure 3. Determine the acceleration of block B and the angular acceleration of the link at this instant.

(Marks: 15)

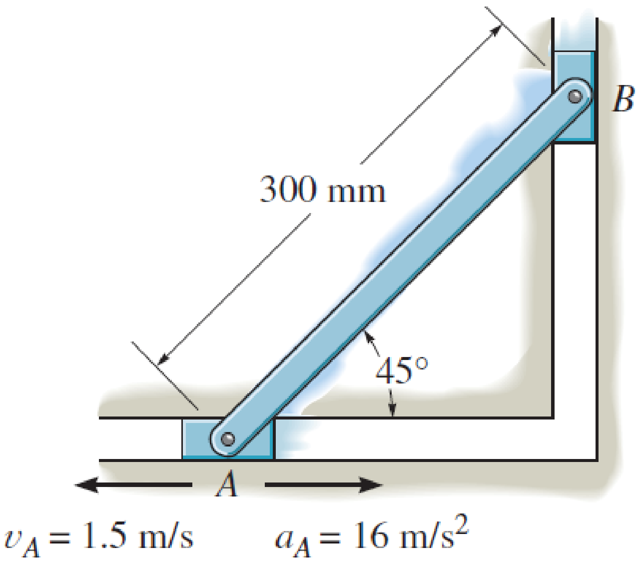



Figure 3**

***Hibbeler R.C.: Engineering Mechanics, Dynamics, Thirteen Edition, Pearson Prentice Hall, 2013*

## Question 4

The system is released from rest with the cable taut, and the homogeneous cylinder does not slip on the rough incline. Determine the angular acceleration of the cylinder and the minimum coefficient $\mu_s$ of friction for which the cylinder will not slip.

(Marks: 20)

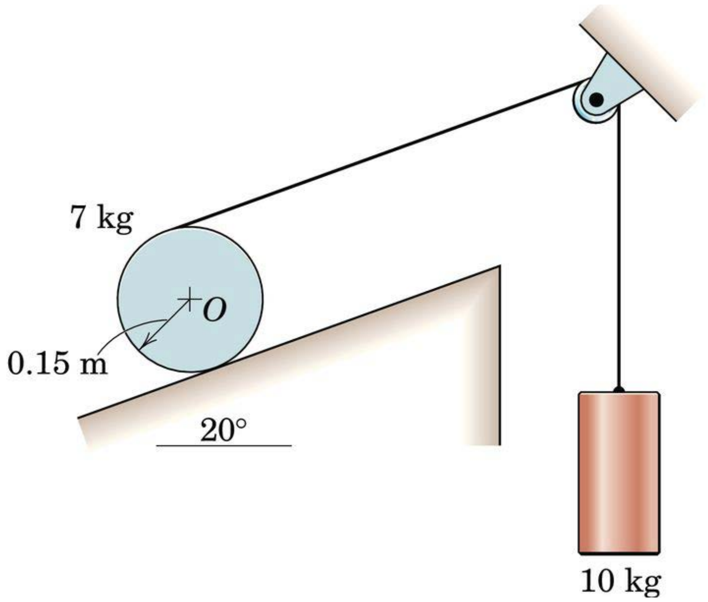



Figure 4*

**Meriam J.L., Kraige L.G.: Engineering Mechanics, Volume 2 Dynamics, Eighth Edition, John Wiley and Sons, 2015*

## Question 5

The uniform slender bar of mass $m$ = 1.196 kg pivots freely about a horizontal axis through $O$. The spring constant $k$ is 200 N/m and the distance $b$ is 200 mm. Determine the angular velocity $\omega$ of the bar when the angular displacement is 15° clockwise if the bar is released from rest in the horizontal position shown on Figure 5 where the spring is unstretched.

(Marks: 20)

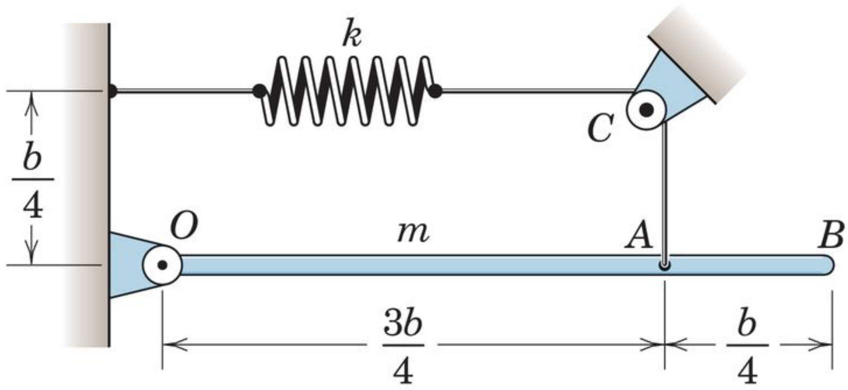



Figure 5*

**Meriam J.L., Kraige L.G.: Engineering Mechanics, Volume 2 Dynamics, Eighth Edition, John Wiley and Sons, 2015*

## Question 6

A massless rope carries two masses of 5 and 7 kg when hanging on a pulley of mass 5 kg, radius 600 mm, and radius of gyration 450 mm. How long will it take to change the speed of the masses from 3 to 6 m/s?

(Marks: 15)

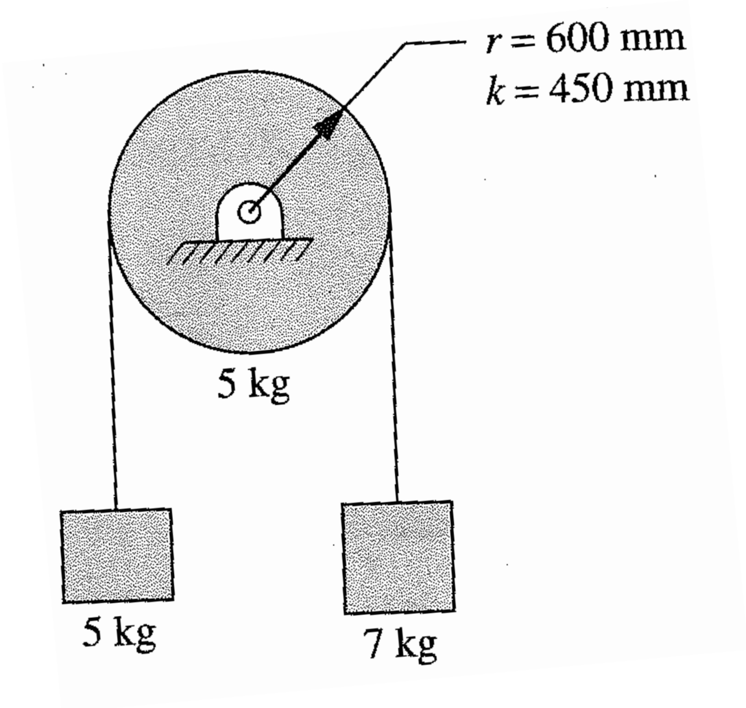



Figure 6***

****Nelson E.W. et al.: Engineering Mechanics Dynamics, 6th Edition, McGraw-Hill, 2011*

## Fundamental Equations of Dynamics

### KINEMATICS

**Particle Rectilinear Motion**

| Variable $a$ | Constant $a = a_c$ |
|---|---|
| $a = \frac{dv}{dt}$ | $v = v_0 + a_c t$ |
| $v = \frac{ds}{dt}$ | $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$ |
| $a\,ds = v\,dv$ | $v^2 = v_0^2 + 2a_c(s - s_0)$ |

**Particle Curvilinear Motion**

| $x, y, z$ Coordinates | $r, \theta, z$ Coordinates |
|---|---|
| $v_x = \dot{x}$ $\quad a_x = \ddot{x}$ | $v_r = \dot{r}$ $\quad a_r = \ddot{r} - r\dot{\theta}^2$ |
| $v_y = \dot{y}$ $\quad a_y = \ddot{y}$ | $v_\theta = r\dot{\theta}$ $\quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ |
| $v_z = \dot{z}$ $\quad a_z = \ddot{z}$ | $v_z = \dot{z}$ $\quad a_z = \ddot{z}$ |

*n, t, b Coordinates*

| | |
|---|---|
| $v = \dot{s}$ | $a_t = \dot{v} = v\frac{dv}{ds}$ |
| | $a_n = \frac{v^2}{\rho} \quad \rho = \frac{[1 + (dy/dx)^2]^{3/2}}{\lvert d^2y/dx^2 \rvert}$ |

*Relative Motion*

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$ $\qquad$ $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

**Rigid Body Motion About a Fixed Axis**

| Variable $\alpha$ | Constant $\alpha = \alpha_c$ |
|---|---|
| $\alpha = \frac{d\omega}{dt}$ | $\omega = \omega_0 + \alpha_c t$ |
| $\omega = \frac{d\theta}{dt}$ | $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$ |
| $\omega\,d\omega = \alpha\,d\theta$ | $\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$ |

*For Point P*

$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$

**Relative General Plane Motion—Translating Axes**

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

**Relative General Plane Motion—Trans. and Rot. Axis**

$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

### KINETICS

**Mass Moment of Inertia** $\quad I = \int r^2\,dm$

*Parallel-Axis Theorem* $\quad I = I_G + md^2$

*Radius of Gyration* $\quad k = \sqrt{\frac{I}{m}}$

**Equations of Motion**

| | |
|---|---|
| Particle | $\Sigma\mathbf{F} = m\mathbf{a}$ |
| Rigid Body (Plane Motion) | $\Sigma F_x = m(a_G)_x$ |
| | $\Sigma F_y = m(a_G)_y$ |
| | $\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$ |

**Principle of Work and Energy**

$T_1 + \Sigma U_{1-2} = T_2$

***Kinetic Energy***

| | |
|---|---|
| Particle | $T = \frac{1}{2}mv^2$ |
| Rigid Body (Plane Motion) | $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$ |

***Work***

| | |
|---|---|
| Variable force | $U_F = \int F\cos\theta\,ds$ |
| Constant force | $U_F = (F_c\cos\theta)\,\Delta s$ |
| Weight | $U_W = -W\Delta y$ |
| Spring | $U_s = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right)$ |
| Couple moment | $U_M = M\Delta\theta$ |

**Power and Efficiency**

$P = \frac{dU}{dt} = \mathbf{F}\cdot\mathbf{v} \qquad \varepsilon = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{U_{\text{out}}}{U_{\text{in}}}$

**Conservation of Energy Theorem**

$T_1 + V_1 = T_2 + V_2$

***Potential Energy***

$V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

**Principle of Linear Impulse and Momentum**

| | |
|---|---|
| Particle | $m\mathbf{v}_1 + \Sigma\int \mathbf{F}\,dt = m\mathbf{v}_2$ |
| Rigid Body | $m(\mathbf{v}_G)_1 + \Sigma\int \mathbf{F}\,dt = m(\mathbf{v}_G)_2$ |

**Conservation of Linear Momentum**

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

**Coefficient of Restitution** $\quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

**Principle of Angular Impulse and Momentum**

| | |
|---|---|
| Particle | $(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ |
| Rigid Body (Plane motion) | $(\mathbf{H}_G)_1 + \Sigma\int \mathbf{M}_G\,dt = (\mathbf{H}_G)_2$ where $H_G = I_G\omega$ |
| | $(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = I_O\omega$ |

**Conservation of Angular Momentum**

$\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$

*Hibbeler R.C.: Engineering Mechanics, Dynamics, Fourteenth Edition, Pearson Prentice Hall, 2016*

## Additional Formulae:

Relative Plane Motion - Translating Axes (B moves along a circular arc with respect to A):

$$\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{B/A}$$

$$\boldsymbol{a}_B = \boldsymbol{a}_A + \boldsymbol{\alpha} \times \boldsymbol{r}_{B/A} - \omega^2 \boldsymbol{r}_{B/A}$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx} \qquad \frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$
$$b^2 = a^2 + c^2 - 2ac \cos \beta$$
$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

Integrals:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

$$\int \frac{dx}{x} = \ln x$$

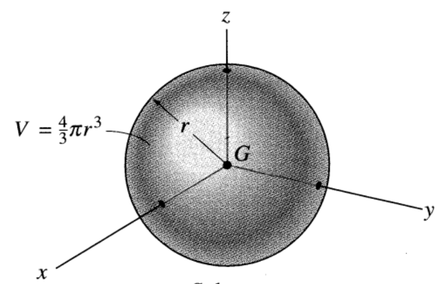
$$\int \sqrt{a + bx} \, dx = \frac{2}{3b} \sqrt{(a + bx)^3}$$

$$\int x\sqrt{a + bx} \, dx = \frac{2}{15b^2} (3bx - 2a) \sqrt{(a + bx)^3}$$

$$\int e^{ax} \, dx = \frac{e^{ax}}{a}$$

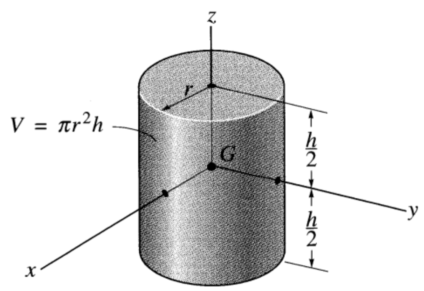
$$\int xe^{ax} \, dx = \frac{e^{ax}}{a^2} (ax - 1)$$

# Center of Gravity and Mass Moment of Inertia of Homogeneous Solids



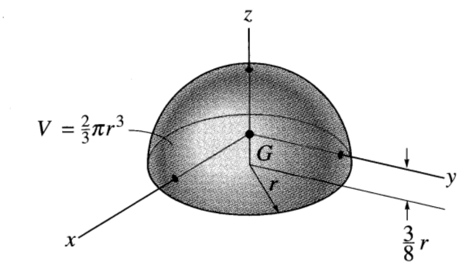

Sphere

$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$



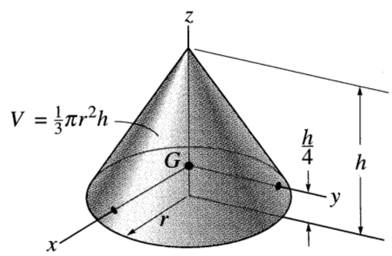

Cylinder

$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2}mr^2$



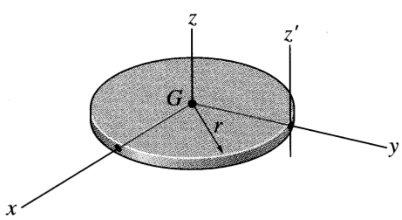

Hemisphere

$I_{xx} = I_{yy} = 0.259mr^2 \quad I_{zz} = \frac{2}{5}mr^2$



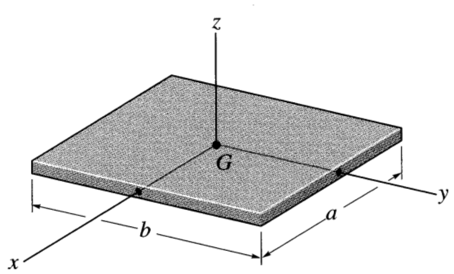

Cone

$I_{xx} = I_{yy} = \frac{3}{80} m(4r^2 + h^2) \quad I_{zz} = \frac{3}{10}mr^2$



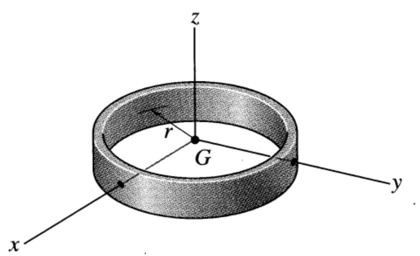

Thin Circular disk

$I_{xx} = I_{yy} = \frac{1}{4} mr^2 \quad I_{zz} = \frac{1}{2}mr^2 \quad I_{z'z'} = \frac{3}{2} mr^2$



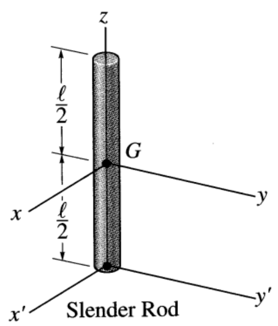

Thin plate

$I_{xx} = \frac{1}{12} mb^2 \quad I_{yy} = \frac{1}{12} ma^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$




Thin ring

$I_{xx} = I_{yy} = \frac{1}{2} mr^2 \quad I_{zz} = mr^2$




Slender Rod

$I_{xx} = I_{yy} = \frac{1}{12} m\ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m\ell^2 \quad I_{z'z'} = 0$

*Hibbeler R.C.: Engineering Mechanics, Dynamics, Fourteenth Edition, Pearson Prentice Hall, 2016*