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Family Name					
Given Name/s					
Student Number					
Teaching Period	Semester 2, 2018				

SMA102 – Mathematics 1B	DURATION	
	Reading Time:	10 minutes
	Writing Time:	180 minutes
INSTRUCTIONS TO CANDIDATES		
<p>1.1 This paper contains six questions. Answer all six (6) questions.</p> <p>1.2 All questions are of equal value, and parts carry marks as indicated.</p> <p>1.3 All symbols, unless stated otherwise, have their usual meanings.</p> <p>1.4 Read ALL questions carefully.</p> <p>1.5 Answers that do not show detailed working will attract little marks.</p>		
EXAM CONDITIONS		
<p><u>You may begin writing from the commencement of the examination session.</u> The reading time indicated above is provided as a guide only.</p>		
This is a CLOSED BOOK examination		
Any non-programmable calculator is permitted		
No handwritten notes are permitted		
No dictionaries are permitted		
ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED	
No additional printed material is permitted	1 x 20 Page Book 1 x Scrap Paper Formula Sheet/s	

**THIS EXAMINATION IS PRINTED
DOUBLE-SIDED.**

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LEFT BLANK.**

Answer ALL questions in the answer booklet provided.

Marks for each question are indicated.

Question 1

- (a) Using integration by parts, evaluate the following indefinite integrals: (Marks: 8)

(i) $\int x^2 \cos x \, dx$

(ii) $\int x \ln x \, dx$

- (b) Using partial fractions decomposition, evaluate the following integral: (Marks: 8)

$$\int \frac{5x - 10}{x^2 - 3x - 4} \, dx$$

- (c) Find the area of the region enclosed by the curve $y = x^2 + 1$, the line $y = x$ in the interval $x = -1$ to $x = 2$. (Marks: 4)

Question 2

- (a) Evaluate the following integrals using appropriate substitutions: (Marks: 7)

(i) $\int x^2 \sqrt{x-1} \, dx$

(ii) $\int \tan 2t \sec^3 2t \, dt$

- (b) Find the exact arc length of the curve (Marks: 6)

$$y = x^{2/3}$$

over the interval $x = 0$ to $x = 8$.

- (c) (i) Sketch the region enclosed between the curves $x = y^2$ and $y = x - 2$. (Marks: 3)

(ii) Use the cylindrical shells method to determine the volume of the solid generated when the region enclosed by $x = 2y - y^2$ and $x = 0$ is revolved about the x -axis.

(Marks: 4)

Question 3

- (a) By revolving the semicircle given by: (Marks: 8)

$$x = r \cos t, y = r \sin t, (0 \leq t \leq \pi)$$

about the x-axis, derive the surface area of the sphere thus generated.

- (b) Using the divergence theorem, determine whether the following series converge: (Marks: 5)

$$(i) \sum_{k=1}^{\infty} \frac{k^2 + k + 3}{2k^2 + 1} \qquad (ii) \sum_{k=1}^{\infty} \frac{1}{5k + 2}$$

- (c) Using the theorem for determining the sum of two series, find the sum of the following series: (Marks: 7)

$$\left(\frac{1}{2} + \frac{1}{4}\right) + \left(\frac{1}{2^2} + \frac{1}{4^2}\right) + \dots + \left(\frac{1}{2^k} + \frac{1}{4^k}\right) + \dots$$

Question 4

- (a) Find the Taylor series of $f(x) = \frac{1}{x+2}$ at $x_0 = 3$, and write it in sigma notation. (Marks: 7)

- (b) Use the method of substitution to evaluate the following improper integral: (Marks: 6)

$$\int_0^{+\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$$

- (c) Find the volume of the solid that is generated when the region enclosed by the curves $y = \sqrt{25 - x^2}$ and $y = 3$ is revolved about the x-axis. (Marks: 7)

Question 5

- (a) Find the standard matrix for the linear transformation T defined by the following

formulas:

(Marks: 6)

(i) $T(x_1, x_2) = (2x_1 - x_2, x_1 + x_2)$

(ii) $T(x_1, x_2, x_3, x_4) = (7x_1 + 2x_2 - x_3 + x_4, x_2 + x_3, -x_1)$

- (b) Find the Euclidean inner product of the following vectors:

(Marks: 6)

(i) $\mathbf{u} = (3, 1, 4, -5)$ and $\mathbf{v} = (2, 2, -4, -3)$, find $\mathbf{u} \cdot \mathbf{v}$ in R^4

(ii) $\mathbf{a} = 3\mathbf{u} + 2\mathbf{v}$ and $\mathbf{b} = 4\mathbf{u} + 7\mathbf{v}$, find $\mathbf{a} \cdot \mathbf{b}$ in R^4

- (c) Use the Gram-Schmidt process to transform the basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ in R^3 into an orthonormal basis, where $\mathbf{u}_1 = (1, 1, 1)$, $\mathbf{u}_2 = (-1, 1, 0)$ and $\mathbf{u}_3 = (1, 2, 1)$.

(Marks: 8)

Question 6

- (a) Find the rank and nullity of matrix A given by:

(Marks: 7)

$$\begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

- (b) The eigenvalues (λ) of matrix A given by:

(Marks: 8)

$$A = \begin{bmatrix} 6 & 2\sqrt{3} \\ 2\sqrt{3} & 7 \end{bmatrix}$$

are $\lambda = 3$ and $\lambda = 10$. Find a matrix P that can diagonalise A and calculate $P^{-1}AP$.

- (c) Consider the vectors $\mathbf{u} = (1, 2, -1)$ and $\mathbf{v} = (6, 4, 2)$ in R^3 . Show that the vector $\mathbf{w} = (9, 2, 7)$ is a linear combination of \mathbf{u} and \mathbf{v} .

(Marks: 5)