SMA 102 guiding solutions: 2018 Final exam

Question 1 a)

(i) For integration by parts use $\int udv = uv - \int vdu$

$$\int x^2 \cos x \, dx$$

Let $u = x^2, du = 2xdx, dv = \cos x \, dx, v = \sin x$

$$\int x^2 \cos x \, dx = x^2 \sin x - \int \sin x \,(2xdx)$$

$$= x^2 \sin x - 2\int x \sin x \, dx$$

Apply integration by parts again

Let $u = x, du = dx,\ dv = \sin x \, dx, v = -\cos x$

$$= x^2 \sin x - 2[x(-\cos x) - \int -\cos x \, dx]$$

$$= x^2 \sin x - 2[-x\cos x + \int \cos x \, dx]$$

$$= x^2 \sin x - 2[-x\cos x + \sin x] + c$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + c$$

(ii)

We let

$$u = \ln x, \qquad \therefore \frac{du}{dx} = \frac{1}{x}$$

$$dv = x\,dx \qquad \therefore v = \frac{x^2}{2}$$

Then

$$\int x \ln x \, dx = \ln x \frac{x^2}{2} - \int\left(\frac{x^2}{2}\right)\left(\frac{1}{x}\right)dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\,dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$$

1 b)

using partial fractions;

$$\frac{5x-10}{x^2-3x-4} = \frac{5x-10}{(x-4)(x+1)} = \frac{A}{(x-4)} + \frac{B}{(x+1)}$$

$$\frac{5x-10}{(x-4)(x+1)} = \frac{A(x+1)+B(x-4)}{(x-4)(x+1)}$$

$$\therefore A(x+1)+B(x-4) = 5x-10$$

$$Ax + A + Bx - 4B = 5x - 10$$

equating coefficients:

$$A + B = 5$$

$$A - 4B = -10$$

solve for A and B:

$$B = 3, A = 2,$$

$$\frac{5x-10}{x^2-3x-4} = \frac{2}{(x-4)} + \frac{3}{(x+1)}$$

$$\therefore \int \frac{5x-10}{x^2-3x-4}\,dx = \int \frac{2}{(x-4)}\,dx + \int \frac{3}{(x+1)}\,dx$$

$$= 2\ln|x-4| + 3\ln|x+1| + c$$

1 c)

$$A = \int_a^b (f(x) - g(x))\,dx$$

$$A = \int_{-1}^{2} (x^2 + 1 - x)\,dx$$

$$A = \left[\frac{x^3}{3} + x - \frac{x^2}{2}\right]_{-1}^{2}$$

$$A = \left(\frac{2^3}{3} + 2 - \frac{2^2}{2}\right) - \left(\frac{(-1)^3}{3} + (-1) - \frac{(-1)^2}{2}\right) = \frac{27}{6} = \frac{9}{2}$$

Question 2 a)

(i)

Solution. The composition $\sqrt{x-1}$ suggests the substitution

$$u = x - 1 \quad \text{so that} \quad du = dx$$

From the first equality in (4)

$$x^2 = (u+1)^2 = u^2 + 2u + 1$$

so that

$$\int x^2\sqrt{x-1}\,dx = \int (u^2+2u+1)\sqrt{u}\,du = \int (u^{5/2} + 2u^{3/2} + u^{1/2})\,du$$

$$= \tfrac{2}{7}u^{7/2} + \tfrac{4}{5}u^{5/2} + \tfrac{2}{3}u^{3/2} + C$$

$$= \tfrac{2}{7}(x-1)^{7/2} + \tfrac{4}{5}(x-1)^{5/2} + \tfrac{2}{3}(x-1)^{3/2} + C \blacktriangleleft$$

(ii)

$$\int \tan 2t \sec^3 2t\,dt$$

let $2t = x$

$$\therefore\ dt = \frac{1}{2}dx$$

$$I = \frac{1}{2}\int \tan x \sec^3 x\,dx$$

Rewrite as

$$I = \frac{1}{2}\int \tan x \sec x \sec^2 x\,dx$$

Substitute $u = \sec x$, $du = \sec x \tan x$

This gives:

$$I = \frac{1}{2}\int u^2\,du$$

$$= \frac{1}{2}\left[\frac{1}{3}u^3\right] + C$$

$$= \frac{1}{2}\left[\frac{\sec^3 x}{3}\right] + C$$

$$I = \frac{1}{6}\sec^3(2t) + C$$

b) $y = x^{2/3}, \frac{dy}{dx} = \frac{2}{3}x^{-1/3} \to \infty$ as $x \to 0$ therefore use $x = y^{3/2}$

to write $\frac{dx}{dy} = \frac{3}{2}y^{1/2}$

When $= 0$, $y = 0$

$x = 8$, $y = (8^{2/3}) = 4.$

$$L = \int_a^b \sqrt{(dx)^2 + (dy)^2} = \int_a^b \sqrt{1 + (dx/dy)^2}\, dy$$

$$L = \int_0^4 \sqrt{1 + \left(\frac{3}{2}y^{1/2}\right)^2}\, dy = \int_0^4 \sqrt{1 + \frac{9}{4}y\, dy}$$

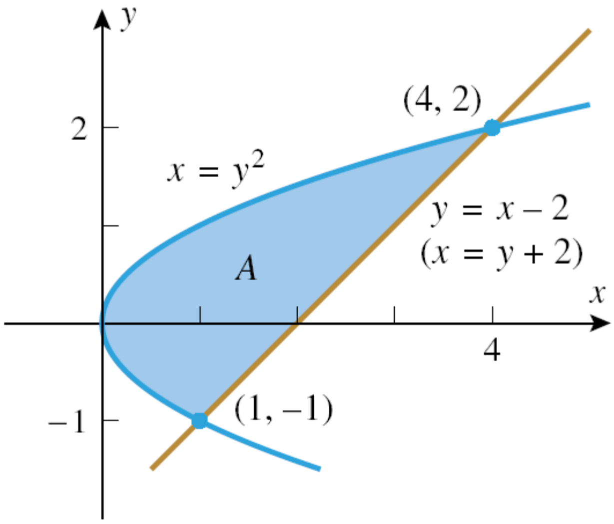
Let $t = 1 + \frac{9}{4}y, dt = \frac{9}{4}dy$

When $y = 0, t = 1$ and $y = 4, t = 10$

$$L = \int_1^{10} \sqrt{t}\frac{4}{9}dt = \frac{4 * 2}{9 * 3}\left[t^{3/2}\right]_1^{10}$$

$$L = \frac{8}{27}(10\sqrt{10} - 1)$$

c) i

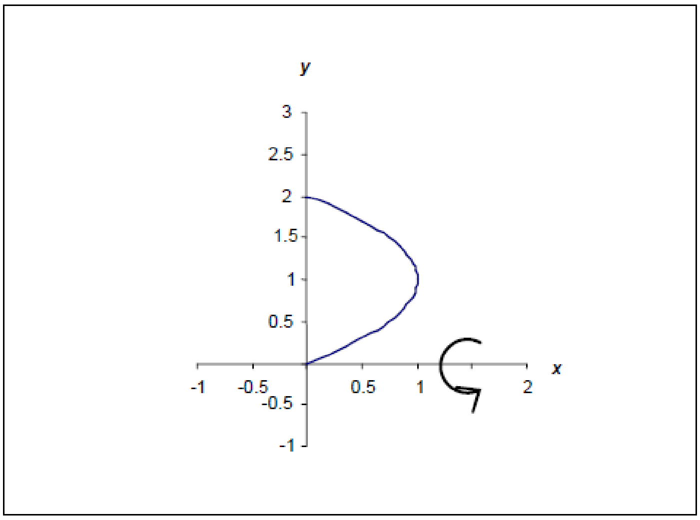


(a)

(ii)

$$x = 2y - y^2 = y(2-y)$$

$\therefore\ x = 0$ when $y(2-y) = 0$ or at $y = 0$ and $y = 2$.



$$V = 2\pi\int_0^2 y(2y - y^2)\,dy$$

$$= 2\pi\int_0^2 (2y^2 - y^3)\,dy$$

$$= 2\pi\left[\frac{2}{3}y^3 - \frac{1}{4}y^4\right]_0^2$$

$$= 2\pi\left[\frac{2}{3}2^3 - \frac{1}{4}2^4\right]_0^2$$

$$= 2\pi\left[\frac{16}{3} - \frac{16}{4}\right]$$

$$= 32\pi\left[\frac{1}{3} - \frac{1}{4}\right]$$

$$= 32\pi\left[\frac{4}{12} - \frac{3}{12}\right]$$

$$= \frac{32\pi}{12}$$

$$V = \frac{8\pi}{3} \text{ cubic units}$$

Question 3

$$\text{(a)}\quad S = 2\pi\int_a^b f(x)\sqrt{1+[f'(x_k)]^2}\,dx$$

$$= 2\pi\int_a^b y(t)\sqrt{[x'(t)]^2+[y'(t)]^2}\,dt$$

$x = r\cos t,\quad y = r\sin t,\quad (0 \le t \le \pi)\quad$ about x-axis

$x'(t) = -r\sin t,\ y'(t) = r\cos t,$

$$S = 2\pi\int_0^\pi r\sin t\sqrt{[-r\sin t]^2+[r\cos t]^2}\,dt$$

$$= 2\pi r\int_0^\pi \sin t\sqrt{r^2\sin^2 t + r^2\cos^2 t}\,dt$$

$$= 2\pi r\int_0^\pi \sin t\sqrt{r^2(\sin^2 t+\cos^2 t)}\,dt$$

$$= 2\pi r\int_0^\pi r\sin t\,dt$$

$$= 2\pi r^2[-\cos t]_0^\pi$$

$$= 2\pi r^2[-\cos\pi+\cos 0]$$

$$= 2\pi r^2(1+1)$$

$$S = 4\pi r^2$$

3 b)

The Divergence Test

$\lim_{k\to+\infty} u_k \neq 0$, then the series $\sum u_k$ diverges

$\lim_{k\to+\infty} u_k = 0$; then the series $\sum u_k$ may either converge or diverge.

i) $$\sum_{k=1}^{\infty}\frac{k^2+k+3}{2k^2+1}$$

$$u_k = \frac{k^2+k+3}{2k^2+1}$$

$$\lim_{k\to+\infty}\frac{1+\frac{1}{k}+\frac{3}{k^2}}{2+\frac{1}{k^2}} = \frac{1}{2} \neq 0$$

The series diverges.

ii) $\sum_{k=1}^{\infty} \frac{1}{5k+2}$

$$\lim_{k \to +\infty} \frac{1}{5k+2} = 0$$

Hence series may converge or diverge.

Now apply the integral test:

$$\sum_{k=1}^{\infty} \frac{1}{5k+2} = \int_{1}^{\infty} \frac{1}{5x+2}\,dx = \frac{1}{5}\left[\ln(5x+2)\right]_{1}^{\infty} \Rightarrow \infty$$

Hence the series diverges.

3 c)

$$\left(\frac{1}{2}+\frac{1}{4}\right)+\left(\frac{1}{2^2}+\frac{1}{4^2}\right)+\ldots+\left(\frac{1}{2^k}+\frac{1}{4^k}\right)+\ldots$$

$$= \sum_{k=1}^{\infty}\left(\frac{1}{2^k}+\frac{1}{4^k}\right)$$

The series can be written as the sum of two series:

$$= \sum_{k=1}^{\infty}\frac{1}{2^k} + \sum_{k=1}^{\infty}\frac{1}{4^k}$$

Both are geometric series.

Series (1).

$\sum_{k=1}^{\infty}\frac{1}{2^k}$ is a geometric series with $a = \frac{1}{2}$, $r = \frac{1}{2}$

$$\therefore S = \frac{a}{1-r} = \frac{1/2}{1-1/2} = 1$$

Series (2).

$\sum_{k=1}^{\infty}\frac{1}{4^k}$ is a geometric series with $a = \frac{1}{4}$, $r = \frac{1}{4}$

$$\therefore S = \frac{a}{1-r} = \frac{1/4}{1-1/4} = \frac{1}{3}$$

Total sum = $1 + \frac{1}{3} = \frac{4}{3}$

Question 4

a) Taylor Series of $f(x)$:

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \\ + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

Here $x_0 = 3$

$$f(x) = (x+2)^{-1}$$
$$f(3) = \frac{1}{5} = 5^{-1}$$

$$f'(x) = -(x+2)^{-2}$$
$$f'(3) = -5^{-2}$$

$$f''(x) = 2(x+2)^{-3} = 2!(x+2)^{-3}$$
$$f''(3) = 2!5^{-3}$$

$$f'''(x) = -3!(x+2)^{-4}$$
$$f'''(3) = -3!5^{-4}$$

$\therefore \frac{1}{(x+2)}$ in Taylor Series at $x_0 = 3$ is:

$$\frac{1}{(x+2)} = \frac{1}{5} - \frac{1}{5^2}(x-3) + \frac{2!}{5^3}\frac{(x-3)^2}{2!} + \frac{3!}{5^4}\frac{(x-3)^3}{3!} + \cdots$$

$$= \frac{1}{5} - \frac{1}{5^2}(x-3) + \frac{(x-3)^2}{5^3} - \frac{(x-3)^3}{5^4} + \cdots$$

$\therefore$ in Sigma notation:

$$\frac{1}{(x+2)} = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{(x-3)^{k-1}}{5^k}$$

4 b)

$$\int_{0}^{+\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx = \lim_{b\to+\infty} \int_{0}^{b} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$$

At $x = 0$, the integral becomes infinite therefore it may be written as

$$\lim_{b\to+\infty, a\to 0} \int_{a}^{b} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$$

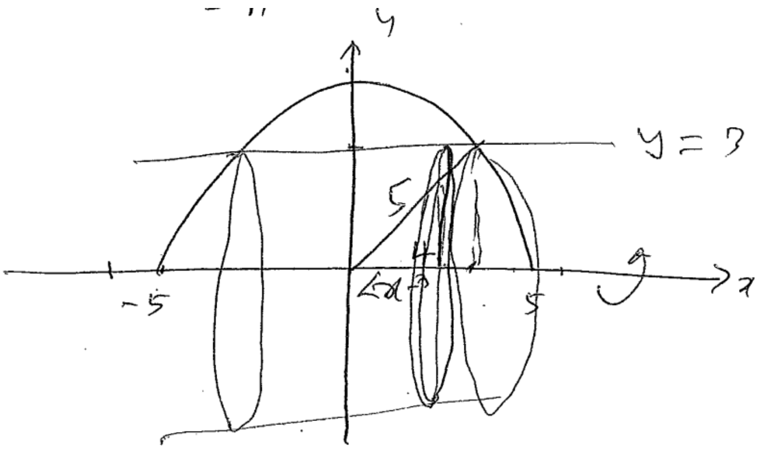
Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}} dx$

As $x \to 0$, $u \to 0$ and $x \to +\infty$, $u \to +\infty$, therefore the integral becomes

$$= 2 \lim_{b\to+\infty, a\to 0} \int_{a}^{b} \frac{e^{-u}}{u} u du = 2 \lim_{b\to+\infty, a\to 0} \int_{a}^{b} e^{-u} du$$

$$= -2 \lim_{b\to+\infty, a\to 0} (e^{-u})_{a}^{b} = -2 \left(\lim_{b\to+\infty} e^{-b} - \lim_{a\to 0} e^{-a} \right) = 2$$

4 c)



The x- coordinate of the intersection of the curves is $y = \sqrt{25 - x^2} = 3$

$25 - x^2 = 9 \to \ x = 4$

$V = \int_0^4 \pi y^2 dx = \pi \int_0^4 (25 - x^2) dx = \pi [25x - x^3/3]_0^4$

$= \pi \left(25 \times 4 - \frac{4^3}{3}\right) - \pi \times 0 = \frac{256\pi}{3}$ units

Question 5a)

i) $T(x_1,x_2) = (2x_1 - x_2, x_1 + x_2)$

This transformation is $R^2 \to R^2$ as:

$w_1 = 2x_1 - x_2,$

$w_2 = x_1 + x_2$

$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$ standard matrix

ii) $T(x_1,x_2,x_3,x_4) = (7x_1 + 2x_2 - x_3 + x_4, x_2 + x_3, -x_1)$

The transformation is $R^4 \to R^3$:

$$T(x_1,x_2,x_3,x_4) = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 7 & 2 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$T(x_1,x_2,x_3,x_4) = A\mathbf{x}$

$\therefore$ standard matrix $A = \begin{bmatrix} 7 & 2 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$

5b)

$\mathbf{u} = (3, 1, 4, -5)$, $\mathbf{v} = (2, 2, -4, -3)$

$\mathbf{u} \cdot \mathbf{v} = (3 \times 2) + (1 \times 2) + (4 \times -4) + (-5 \times -3)$
$= 6 + 2 - 16 + 15$
$= 7$

$\mathbf{a} = 3\mathbf{u} + 2\mathbf{v} = (9, 3, 12, -15) + (4, 4, -8, -6)$
$= (13, 7, 4, -21)$

$\mathbf{b} = 4\mathbf{u} + 7\mathbf{v} = (12, 4, 16, -20) + (14, 14, -28, -21)$
$= (26, 18, -12, -41)$

$\mathbf{a} \cdot \mathbf{b} = (13 \times 26) + (7 \times 18) + (4 \times -12) + (-21 \times -41)$
$= 338 + 126 - 48 + 861$
$= 1277$

5c)

$\mathbf{u}_1 = (1, 1, 1), \qquad \mathbf{u}_2 = (-1, 1, 0), \qquad \mathbf{u}_3 = (1, 2, 1)$

$\langle \mathbf{u}_1, \mathbf{u}_2 \rangle = -1 + 1 + 0 = 0$

$\langle \mathbf{u}_1, \mathbf{u}_3 \rangle = 1 + 2 + 1 = 4 \neq 0$

$\langle \mathbf{u}_2, \mathbf{u}_3 \rangle = -1 + 2 + 0 = 1 \neq 0$

These vectors are not orthogonal.

Let us assume that the orthogonal vectors are $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$

Step 1: Set $\mathbf{v}_1 = \mathbf{u}_1$

Step 2: $\mathbf{v}_2 = \mathbf{u}_2 - \frac{\langle \mathbf{u}_2, \mathbf{v}_1 \rangle}{\|\mathbf{v}_1\|^2}\mathbf{v}_1 = \mathbf{u}_2$, since $\langle \mathbf{u}_2, \mathbf{v}_1 \rangle = 0$

Step 3:
$$\mathbf{v}_3 = \mathbf{u}_3 - \frac{\langle \mathbf{u}_3, \mathbf{v}_1 \rangle}{\|\mathbf{v}_1\|^2}\mathbf{v}_1 - \frac{\langle \mathbf{u}_3, \mathbf{v}_2 \rangle}{\|\mathbf{v}_2\|^2}\mathbf{v}_2$$
$$= \mathbf{u}_3 - \frac{4}{\|\mathbf{v}_1\|^2}\mathbf{v}_1 - \frac{1}{\|\mathbf{v}_2\|^2}\mathbf{v}_2$$

$\|\mathbf{v}_1\|^2 = 1 + 1 + 1 = 3$

$\|\mathbf{v}_2\|^2 = 1 + 1 = 2$

$$\therefore \mathbf{v}_3 = \mathbf{u}_3 - \frac{4}{3}\mathbf{v}_1 - \frac{1}{2}\mathbf{v}_2$$
$$= (1, 2, 1) - \frac{4}{3}(1, 1, 1) - \frac{1}{2}(-1, 1, 0)$$
$$= \left(\left(1 - \frac{4}{3} + \frac{1}{2}\right), \left(2 - \frac{4}{3} - \frac{1}{2}\right), \left(1 - \frac{4}{3}\right)\right)$$
$$= \left(\frac{1}{6}, \frac{1}{6}, -\frac{1}{3}\right)$$

These form an orthogonal set.

$\therefore \mathbf{v}_1 = (1, 1, 1),$

$\mathbf{v}_2 = (-1, 1, 0),$

$\mathbf{v}_3 = \left(\frac{1}{6}, \frac{1}{6}, -\frac{1}{3}\right)$

The orthonormal set would be:

$$\mathbf{v}_1 = \frac{\mathbf{v}_1}{\|\mathbf{v}_1\|} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$\mathbf{v}_2 = \frac{\mathbf{v}_2}{\|\mathbf{v}_2\|} = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$$\mathbf{v}_3 = \frac{\mathbf{v}_3}{\|\mathbf{v}_3\|} = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{\sqrt{2}}{\sqrt{3}}\right)$$

Question 6a)

$$\text{a)} = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

Reduce it to reduced row-echelon form:

Row 3 + Row 1

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ 0 & 7 & 7 & 4 \end{bmatrix}$$

Row 2 − 2 × Row 1

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & -7 & -7 & -4 \\ 0 & 7 & 7 & 4 \end{bmatrix}$$

Row 3 + Row 2

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & -7 & -7 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row 2 ÷ − 7

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 1 & \frac{4}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row 1 − 4 × Row 2

$$A = \begin{bmatrix} 1 & 0 & 1 & -\frac{2}{7} \\ 0 & 1 & 1 & \frac{4}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is in reduced row-echelon form.

The common row and column dimensions = 2

$\therefore$ rank(A) = 2

Nullity:

$A\mathbf{x} = \mathbf{0}$

$$x_1 + x_3 - \frac{2}{7} x_4 = 0$$

$$x_2 + x_3 + \frac{4}{7} x_4 = 0$$

4 unknowns and 2 equations. That means 2 free variables.

Hence nullity(A) = 2.

6b) Using the given eigenvalues, solve for the eigenvectors

When $\lambda = 3$

$$(3I - A) = \begin{pmatrix} 3-6 & -2\sqrt{3} \\ -2\sqrt{3} & 3-7 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -3 & -2\sqrt{3} \\ -2\sqrt{3} & -4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-3x_1 - 2\sqrt{3}x_2 = 0$, $-2\sqrt{3}x_1 - 4x_2 = 0$, if $x_2 = s$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -2/\sqrt{3}s \\ s \end{pmatrix} = s\begin{pmatrix} -2/\sqrt{3} \\ 1 \end{pmatrix}. \ |\mathbf{x}_1| = \sqrt{(-2/\sqrt{3})^2 + (1)^2} = \sqrt{\frac{7}{3}}$$ Normalizing the vectors

$$\mathbf{u}_1 = \frac{\mathbf{x}_1}{|\mathbf{x}_1|} = \begin{pmatrix} -2/\sqrt{7} \\ \sqrt{3}/\sqrt{7} \end{pmatrix},$$

When $\lambda = 10$

$$(10I - A) = \begin{pmatrix} 10-6 & -2\sqrt{3} \\ -2\sqrt{3} & 10-7 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 & -2\sqrt{3} \\ -2\sqrt{3} & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$4x_1 - 2\sqrt{3}x_2 = 0$, $-2\sqrt{3}x_1 + 3x_2 = 0$, if $x_2 = s$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2}s \\ s \end{pmatrix} = s\begin{pmatrix} \frac{\sqrt{3}}{2} \\ 1 \end{pmatrix}. |\mathbf{x}_1| = \sqrt{(\sqrt{3}/2)^2 + (1)^2} = \sqrt{\frac{7}{4}} = \frac{1}{2}\sqrt{7}$$

$$\mathbf{u}_2 = \frac{\mathbf{x}_2}{|\mathbf{x}_2|} = \begin{pmatrix} \sqrt{3}/\sqrt{7} \\ 2/\sqrt{7} \end{pmatrix}$$

$$P = \begin{bmatrix} -2/\sqrt{7} & \sqrt{3}/\sqrt{7} \\ \sqrt{3}/\sqrt{7} & 2/\sqrt{7} \end{bmatrix}, \ P^{-1} = \frac{1}{-1}\begin{bmatrix} 2/\sqrt{7} & -\sqrt{3}/\sqrt{7} \\ -\sqrt{3}/\sqrt{7} & -2/\sqrt{7} \end{bmatrix} = \begin{bmatrix} -2/\sqrt{7} & \sqrt{3}/\sqrt{7} \\ \sqrt{3}/\sqrt{7} & 2/\sqrt{7} \end{bmatrix},$$

$$P^{-1}AP = \begin{bmatrix} 3 & 0 \\ 0 & 10 \end{bmatrix}$$

6c)

$\mathbf{u} = (1, 2, -1)$, $\mathbf{v} = (6, 4, 2)$

For proving linear combination for $\mathbf{w} = (9, 2, 7)$, we write $\mathbf{w}$ as:

$\mathbf{w} = k_1\mathbf{u} + k_2\mathbf{v}$

$(9, 2, 7) = k_1(1, 2, -1) + k_2(6, 4, 2)$

Equate the corresponding components to get:

$k_1 + 6k_2 = 9$ ________ (1)

$2k_1 + 4k_2 = 2$ ________ (2)

$-k_1 + 2k_2 = 7$ ________ (3)

Add (1) and (3)

$8k_2 = 16$
$k_2 = 2$

Substitute $k_2 = 2$ into (1), this gives

$k_1 + 12 = 9$

$k_1 = -3$

Check by substitution into (2), this gives

$2k_1 + 4k_2 = (2 \times -3) + (4 \times 2) = -6 + 8 = 2$ ✓

$\therefore\ \mathbf{w} = -3\mathbf{u} + 2\mathbf{v}$