Charles Darwin University

Final Examination

Family Name						
Given Name/s						
Student Number						
Teaching Period	Semester 1, 2019					

STA510 – Business Statistics	DURATION	
	Reading Time:	10 minutes
	Writing Time:	180 minutes

INSTRUCTIONS TO CANDIDATES

Section A: 20 Multiple Choice Questions Marks: 10 Time suggested: 50 minutes
Section B: 4 Structured Answer Questions Marks: 40 Time suggested: 130 minutes

Section A	Section B: Q1	Section B: Q2	Section B: Q3	Section B: Q4	Total

EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a RESTRICTED OPEN BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

Any hard copy, unannotated English dictionary is permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
No additional printed material is permitted	1 x 5-Multiple Choice Answer Sheet 1 x Scrap Paper College Multiple Choice Answer Sheet Formula Sheet/s Statistical Table/s

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

THIS PAGE HAS BEEN INTENTIONALLY LEFT BLANK.

Section A

Multiple Choice Questions

Total marks for this section: 10

Suggested time allocation for this Section: 50 minutes

Answer *ALL* Questions

This section should be answered in the Multiple Choice Answer Sheet provided.
Each question is worth 0.5 mark.

Section B

Structured Answer Questions

Total marks for this section: 40

Suggested time allocation for this Section: 180 minutes

Answer All Questions

This section should be answered in the space given in the Exam Paper.
Marks for each question are indicated.

QUESTION 1: PART A *[4 marks]*

The owner of a hardware shop that sells electrical wire by the meter is considering selling the wire in pre-cut lengths in order to reduce labour costs. A sample of lengths (in metres) of wires sold over the course of one day produced the following data:

3, 7, 4, 2.5, 3, 20, 5, 5, 15, 3.5, 3

(i) Find the mean length of wires in the sample **in metres**.

(ii) Find the median and mode length of wires in the sample **in metres**.

QUESTION 1 (continues on next page)

(iii) Determine the sample standard deviation of the length of wires in the sample **in metres**.

(iv) Find the coefficient of variation of the length of wires **expressed as a percentage** (round to two decimal places).

(v) Find the 60^{th} percentile of the length of wires.

QUESTION 1 (continues on next page)

QUESTION 1: PART B [***6 marks]***

The amount of time devoted to studying statistics each week by students who achieve a grade of HD in the unit is a normally distributed random variable with a population mean (μ) of 7.5 hours and a standard deviation (σ) of 2.1 hours.

(i) Find the probability that an HD grade student spends more than 10 hours studying?

(ii) Find the probability that an HD grade student spends between 7 and 9 hours studying?

(iii) Find the probability that an HD grade student spends less than 3 hours studying?

(iv) What is the amount of time below which only 5% of all HD grade students spend studying?

QUESTION 2 [*10 marks]*

To determine the mean waiting time for his customers, a bank manager took a random sample of 50 customers and found that the mean waiting time was 7.2 minutes. Assuming that the population standard deviation is known to be 5 minutes, find the 90% confidence interval estimate of the mean waiting time for all of the bank's customers.

(i) Write down the parameter of interest, its point estimator and the associated sampling distribution (how do you know this?) which would be used for the estimation of the confidence interval.

Parameter of interest: _____

Point estimator: _____

Sampling distribution of the point estimator: _____

(ii) Specify the formula for the 90% confidence interval estimator for the parameter.

(iii) Perform the necessary calculations and write down the lower and upper limits of the 90% confidence interval.

Answer:

Lower limit = _____

Upper limit = _____

Working

QUESTION 2 (continues on next page)

(iv) Interpret the calculated confidence interval.

(v) Describe the effect of increasing the confidence level to 95% on the confidence interval estimate of part (iii). Support your answer with relevant calculations.

QUESTION 3 [10 marks]

Companies that sell groceries over the internet are called e-grocers. Customers enter their orders, pay by credit card, and receive delivery by truck. A potential e-grocer analysed the market and found that for e-grocery business to be profitable, the average order would have to exceed $85. To determine whether an e-grocery would be profitable in one large city, the service was offered to a random sample of 85 customers and the value of orders recorded. The average value of the sample of orders was found to be $89.27 with a standard deviation of $17.30. Does this evidence support at the 5% level of significance an e-grocery will be profitable in this city? Use the following template to arrive at your conclusion.

QUESTION 3 (continues on next page)

Answer

Step 1. Statement of the hypotheses

H_0:

H_A:

Step 2. Test statistic and the standardised test statistic

Step 3. Level of significance

Step 4. Decision rule

Step 5. Computation of the value of the test statistic

Step 6. Conclusion

QUESTION 4: Part A [**6 marks**]

An energy economist is interested in investigating the relationship between electricity consumption in the summer months (in 000 of kilowatt hours) and the number of rooms in a private single family residence. He has selected 10 such residences and found the following information:

Number of Rooms	Electricity Consumption (000 Kilowatt Hours)
14	11
11	8
16	12
8	5
12	10
10	8
12	10
7	6
9	9
12	11

A regression analysis of Y against X using Excel yielded the output below:

SUMMARY OUTPUT								
Regression Statistics								
Multiple R	0.9013							
R Square	0.8124							
Adjusted R Square	0.7889							
Standard Error	1.0387							
Observations	10							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	37.369	37.369	34.638	0.000			
Residual	8	8.631	1.079					
Total	9	46						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.7040	1.447	0.486	0.640	-2.634	4.042	-2.634	4.042
Number of Rooms	0.7474	0.127	5.885	0.000	0.455	1.040	0.455	1.040

QUESTION 4 (continues on next page)

Use the EXCEL results above to answer the following questions.

(i) Write down the estimated regression line ($\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$):

(ii) Interpret the intercept term $\hat{\beta}_0$ of the regression line.

(iii) What information does the slope coefficient provide about the relationship between electricity consumption in the summer months (in 000 of kilowatt hours) and the number of rooms in a private single family residence

(iv) Predict the electricity consumption in the summer months for a 10 room private single family residence.

QUESTION 4 (continues on next page)

(v) Test whether there is a linear relationship exists between electricity consumption in the summer months (in 000 of kilowatt hours) and the number of rooms in a private single family residence at the 5 percent significance level (use the Excel output).

1. Null hypothesis: H_o:

   Alternative hypothesis: H_A:

2. Test statistic and associated sampling distribution:

3. Level of significance: α =

4. Decision rule using p-value method:

5. Conclusion:

(vi) Find is the coefficient of determination (R-squared) and interpret it.

QUESTION 4 (continues on next page)

QUESTION 4: Part B [**4 marks**]

Quarterly sales data belonging to the 40 quarters of the last 10 years (2009-2018) have produced the following trend line and seasonal indexes. Using a multiplicative model, forecast the sales values for the next 4 quarters in 2019.

Trend line: $\hat{y}_t = 150 + 3t$

Year	Quarter	t	$\hat{y}_t$	Seasonal Index (S_{it})	Forecast F_t
2019	1			0.7	
	2			1.2	
	3			1.5	
	4			0.6	

END OF EXAMINATION

Formula List

Summary Measures (n – sample size; N = Population size)

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N} \qquad \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N} \qquad s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

$$\sigma^2 = \frac{\sum_{i=1}^{N} x_i^2 - \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N}}{N} \qquad s^2 = \frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}$$

$$\sigma = \sqrt{\sigma^2} \qquad s = \sqrt{s^2}$$

$$\text{CV} = \frac{\text{Standard Deviation}}{\text{Mean}} \times 100\%$$

Location of the p[th] percentile: $L_p = (n+1)\frac{p}{100}$ IQR = Q_3-Q_1

$$s_{xy} = \frac{1}{n-1}\left(\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}\right); \quad s_x^2 = \frac{1}{n-1}\left(\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right); \quad s_y^2 = \frac{1}{n-1}\left(\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}\right)$$

$$r = \frac{s_{xy}}{\sqrt{s_x^2 s_y^2}} \qquad \hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x; \qquad \hat{\beta}_1 = \frac{s_{xy}}{s_x^2}; \qquad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Probability distributions

Discrete Probability Distribution

$$\mu = E[X] = \sum_i x_i p_i, \quad \sigma^2 = V[X] = \sum_i (x_i - \mu)^2 p_i, \text{ or } \sigma^2 = V[X] = \sum_i x_i^2 p_i - \mu^2$$

Normal Distribution

$$Z = \frac{X - \mu}{\sigma}; \qquad E[\bar{X}] = \mu; \qquad V[\bar{X}] = \frac{\sigma^2}{n}$$

Confidence Intervals

Mean: $\bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ $\bar{X} \pm t_{\alpha/2;\, n-1} \frac{s}{\sqrt{n}}$ $n = \frac{z_{\alpha/2}^2 \sigma^2}{B^2}$

Proportion: $\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ *or* $\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$ $n = \frac{z_{\alpha/2}^2 pq}{B^2}$ *where* $q = 1 - p$

$\hat{p} = \frac{X}{n}$

Hypothesis Testing

Mean: $z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$ $t_{n-1} = \frac{\bar{X} - \mu}{s / \sqrt{n}}$

Proportion: $z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$ *where* $q = 1 - p$

Correlation Analysis and Simple Linear Regression Analysis

Sample regression line

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x; \qquad \hat{\beta}_1 = \frac{s_{xy}}{s_x^2}; \qquad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

SSE and Standard error of estimate

$$SSE = (n-1)\left[s_y^2 - \frac{s_{xy}^2}{s_x^2} \right] \qquad s_e^2 = \frac{SSE}{n-2} \quad or \quad s_e = \sqrt{\frac{SSE}{n-2}}$$

Standard error of the slope and intercept coefficient estimates

$$s_{\hat{\beta}_1} = \frac{s_\varepsilon}{\sqrt{(n-1)s_x^2}} \qquad s_{\hat{\beta}_o} = \sqrt{\frac{s_\varepsilon^2 \sum x_i^2 / n}{(n-1)s_x^2}}$$

Test statistic for the significance of the slope and intercept coefficients

$$t_{n-2} = \frac{\hat{\beta}_0 - \beta_0}{s_{\hat{\beta}_0}} \qquad t_{n-2} = \frac{\hat{\beta}_1 - \beta_1}{s_{\hat{\beta}_1}}$$

Coefficient of determination

$$R^2 = 1 - \frac{SSE}{SST} \qquad SST = (n-1)s_y^2 \qquad R^2 = r^2$$

Coefficient of Correlation

$$r = \sqrt{R^2}$$

Time series forecasting (multiplicative models)

$$F_t = \hat{y}_t . S_{it}$$

Statistical Tables

Normal Distribution

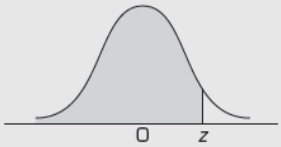
Table 3 Cumulative Standardised Normal Probabilities

$P(-\infty < Z < z)$

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
–3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
–2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
–2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
–2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
–2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
–2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
–2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
–2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
–2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
–2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
–2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
–1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
–1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
–1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
–1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
–1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
–1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
–1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
–1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
–1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
–1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
–0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
–0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
–0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
–0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
–0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
–0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
–0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
–0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
–0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
–0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Normal Distribution (continued)

Table 3 Cumulative Standardised Normal Probabilities (Continued)

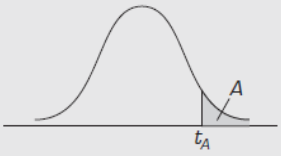


$P(-\infty < Z < z)$

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

t Distribution

Table 4 Critical Values of the Student *t* Distribution



Degrees of freedom	$t_{0.100}$	$t_{0.050}$	$t_{0.025}$	$t_{0.010}$	$t_{0.005}$	Degrees of freedom	$t_{0.100}$	$t_{0.050}$	$t_{0.025}$	$t_{0.010}$	$t_{0.005}$
1	3.078	6.314	12.706	31.821	63.657	29	1.311	1.699	2.045	2.462	2.756
2	1.886	2.920	4.303	6.965	9.925	30	1.310	1.697	2.042	2.457	2.750
3	1.638	2.353	3.182	4.541	5.841	35	1.306	1.690	2.030	2.438	2.724
4	1.533	2.132	2.776	3.747	4.604	40	1.303	1.684	2.021	2.423	2.704
5	1.476	2.015	2.571	3.365	4.032	45	1.301	1.679	2.014	2.412	2.690
6	1.440	1.943	2.447	3.143	3.707	50	1.299	1.676	2.009	2.403	2.678
7	1.415	1.895	2.365	2.998	3.499	55	1.297	1.673	2.004	2.396	2.668
8	1.397	1.860	2.306	2.896	3.355	60	1.296	1.671	2.000	2.390	2.660
9	1.383	1.833	2.262	2.821	3.250	65	1.295	1.669	1.997	2.385	2.654
10	1.372	1.812	2.228	2.764	3.169	70	1.294	1.667	1.994	2.381	2.648
11	1.363	1.796	2.201	2.718	3.106	75	1.293	1.665	1.992	2.377	2.643
12	1.356	1.782	2.179	2.681	3.055	80	1.292	1.664	1.990	2.374	2.639
13	1.350	1.771	2.160	2.650	3.012	85	1.292	1.663	1.988	2.371	2.635
14	1.345	1.761	2.145	2.624	2.977	90	1.291	1.662	1.987	2.368	2.632
15	1.341	1.753	2.131	2.602	2.947	95	1.291	1.661	1.985	2.366	2.629
16	1.337	1.746	2.120	2.583	2.921	100	1.290	1.660	1.984	2.364	2.626
17	1.333	1.740	2.110	2.567	2.898	110	1.289	1.659	1.982	2.361	2.621
18	1.330	1.734	2.101	2.552	2.878	120	1.289	1.658	1.980	2.358	2.617
19	1.328	1.729	2.093	2.539	2.861	130	1.288	1.657	1.978	2.355	2.614
20	1.325	1.725	2.086	2.528	2.845	140	1.288	1.656	1.977	2.353	2.611
21	1.323	1.721	2.080	2.518	2.831	150	1.287	1.655	1.976	2.351	2.609
22	1.321	1.717	2.074	2.508	2.819	160	1.287	1.654	1.975	2.350	2.607
23	1.319	1.714	2.069	2.500	2.807	170	1.287	1.654	1.974	2.348	2.605
24	1.318	1.711	2.064	2.492	2.797	180	1.286	1.653	1.973	2.347	2.603
25	1.316	1.708	2.060	2.485	2.787	190	1.286	1.653	1.973	2.346	2.602
26	1.315	1.706	2.056	2.479	2.779	200	1.286	1.653	1.972	2.345	2.601
27	1.314	1.703	2.052	2.473	2.771	∞	1.282	1.645	1.960	2.326	2.576
28	1.313	1.701	2.048	2.467	2.763						