Charles Darwin University

Final Examination

Family Name	
Given Name/s	
Student Number	
Teaching Period	Semester 1, 2019

ENG421 – Digital Signal Processing	**DURATION**	
	Reading Time:	**10** minutes
	Writing Time:	**120** minutes

INSTRUCTIONS TO CANDIDATES

1. Answer all questions.
2. This exam constitutes 50% of the total marks for this unit.
3. Total number of marks of this exam: 16
   - Question 1 is worth 3 marks
   - Question 2 is worth 2 marks
   - Question 3 is worth 3 marks
   - Question 4 is worth 3 marks
   - Question 5 is worth 2 marks
   - Question 6 is worth 3 marks
4. Note that questions ARE NOT of equal value
5. Read ALL questions carefully
6. Do not commence writing until instructed to do so

EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

Any hard copy, unannotated English dictionary is permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
No additional printed material is permitted	1 x 20 Page Book 1 x Scrap Paper

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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Question 1 (3 marks)

Figure 1 below shows the pole zero plot of a system in the z-domain. The system has three poles and three zeros.

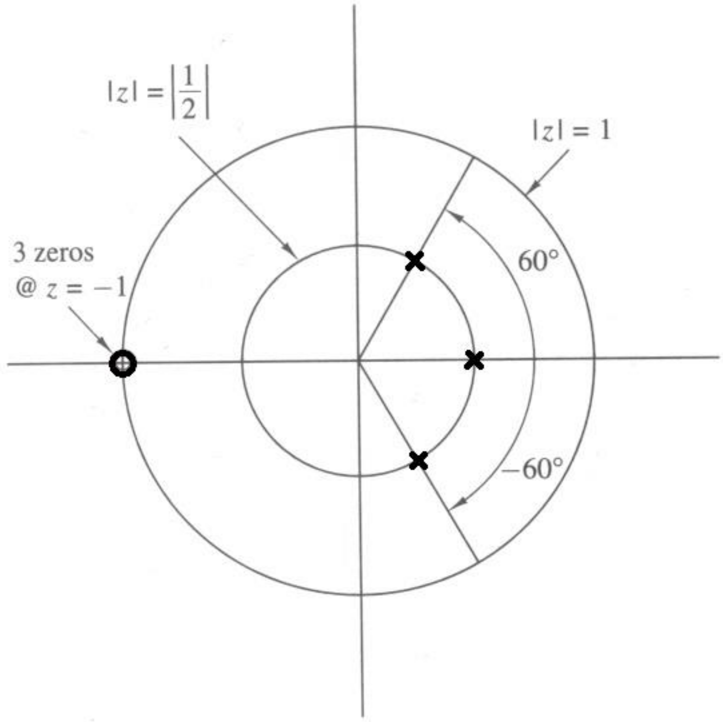

Figure 1, z-domain pole-zero plot of system transfer

Question 1.1 (1 mark)
Given that the static gain (gain at $\hat{\omega} = 0$ [rad]) of the system is 64, determine the z-transform H(z) of the system, using only real coefficients for H(z).

Question 1.2 (1 mark)
Determine the n-domain difference equation of the system, where x[n] is the input of the system and y[n] the output.

Question 1.3 (1 mark)
Calculate and sketch the impulse response for the first 5 data-points $(0 \leq \mathrm{n} \leq 4)$.

Question 2 (2 marks)

A 12 point running average filter is applied to the following two input signals in Question 2.1 and Question 2.2.

Question 2.1 (1 mark)

Determine the response of the filter $y[n]$ to the input signal $x[n]$:

$$x[n] = \delta[n+1] + \delta[n-3]$$

Question 2.2 (1 mark)

Determine the response of the filter $y[n]$ to the input signal $x[n]$:

$$x[n] = \sin\left(\frac{\pi}{2}n - \frac{\pi}{3}\right)$$

Question 3 (3 marks)

A linear time invariant system has the following frequency response:

$$H(\hat{\omega}) = \left(e^{-j\hat{\omega}} \cdot e^{-j\frac{\pi}{2}} - 1\right) \cdot \left(e^{-j\hat{\omega}} \cdot e^{j\frac{\pi}{2}} - 1\right) \cdot \left(2 \cdot e^{-j\hat{\omega}} + 2\right)$$

Question 3.1 (1 mark)

Determine the impulse response h[n] of this system.

Question 3.2 (1 mark)

Sketch the impulse response from n = -5 to n = 5. Describe in words what this linear time invariant system does to the input signal.

Question 3.3 (1 mark)

The input to the system is:

$$x[n] = \delta[n-5] - \frac{1}{5} + 5 \cdot \sin\left(\pi\frac{n}{2} - \frac{\pi}{5}\right)$$

Determine the output y[n] for $-\infty \leq n \leq \infty$.

Question 4 (3 marks)

The following difference equation describes a linear time-invariant filter:

$$y[n+1] = x[n+1] - 2 \cdot x[n] + 0.5 \cdot y[n]$$

Question 4.1 (1 mark)

Determine the system transfer function in the z-domain for this filter. Determine all poles and zeros of this filter.

Question 4.2 (1 mark)

Determine $\left|H(e^{j\hat{\omega}})\right|^2$ for all $\hat{\omega}$.

Question 4.3 (1 mark)

Determine the output of the filter, if the input to the filter is

$$x[n] = 1 - \sin\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{4}n\right)$$

Question 5 (2 marks)

The following difference equation defines an IIR filter:

$$y[n] = x[n] + \frac{1}{4} \cdot y[n-2]$$

Question 5.1 (1 mark)

Determine all poles and zeros of the transfer function $H(z)$ of this IIR filter.

Question 5.2 (1 mark)

Determine the impulse response h[n] of the IIR filter for $-\infty < n < \infty$. Sketch the impulse response from $n =$ -4 to $n = 4$.

Question 6 (3 marks)

A system is defined by a z-domain transfer function with two poles, both at $z = 0$ and two zeros at $z = -1$ and $z = 1$. The gain at $\hat{\omega} = \frac{\pi}{2}$ is 4.

Question 6.1 (1 mark)

Determine the difference equation describing this system, with x[n] as input and y[n] as output.

Question 6.2 (1 mark)

Determine the frequency domain response of this system and derive two simple formulas (without complex terms and without square roots) for the magnitude versus $\hat{\omega}$ and the phase versus $\hat{\omega}$. Make a sketch of the magnitude versus $\hat{\omega}$ and the phase versus $\hat{\omega}$ of the frequency domain response of this system for $0 \leq \hat{\omega} \leq 2\pi$.

Question 6.3 (1 mark)

Determine the output of the system y[n] if the input to the system is

$$x[n] = \sin\left(\frac{2\pi}{3}n + \frac{\pi}{2}\right)$$

Attached Materials *(from J.H. McClellan et al., DSP first, a multimedia approach, Prentice Hall, 1998)*

SHORT TABLE OF z-TRANSFORMS

	$x[n]$		$X(z)$
1.	$ax_1[n] + bx_2[n]$	$\Longleftrightarrow$	$aX_1(z) + bX_2(z)$
2.	$x[n - n_0]$	$\Longleftrightarrow$	$z^{-n_0}X(z)$
3.	$y[n] = x[n] * h[n]$	$\Longleftrightarrow$	$Y(z) = H(z)X(z)$
4.	$\delta[n]$	$\Longleftrightarrow$	1
5.	$\delta[n - n_0]$	$\Longleftrightarrow$	z^{-n_0}
6.	$a^n u[n]$	$\Longleftrightarrow$	$\dfrac{1}{1 - az^{-1}}$

PROCEDURE FOR INVERSE z-TRANSFORMATION ($M < N$)

1. Factor the denominator polynomial of $H(z)$ and express the pole factors in the form $(1 - p_k z^{-1})$ for $k = 1, 2, \ldots, N$.
2. Make a partial fraction expansion of $H(z)$ into a sum of terms of the form

$$H(z) = \sum_{k=1}^{N} \frac{A_k}{1 - p_k z^{-1}} \quad \text{where} \quad A_k = H(z)(1 - p_k z^{-1})\Big|_{z=p_k}$$

3. Write down the answer as

$$h[n] = \sum_{k=1}^{N} A_k (p_k)^n u[n]$$