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Charles Darwin University

Final Examination

Family Name						
Given Name/s						
Student Number						
Teaching Period	Semester 1, 2019					

SMA101 – Mathematics 1A	DURATION	
	Reading Time:	10 minutes
	Writing Time:	180 minutes

INSTRUCTIONS TO CANDIDATES

1 Answer all six questions.
2 All questions are of equal value, and parts carry marks as indicated.
3 Read ALL questions carefully.
4 Show all working neatly in all parts. Answers without working details will attract little marks.
5 All symbols, unless stated otherwise, have their usual meanings.

EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
No additional printed material is permitted	1 x 20 Page Book 1 x Scrap Paper Formula Sheet/s

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Answer ALL questions in the Answer Booklet provided.

Marks for each question are indicated.

Question 1

(a) Solve $|x-3|=5$. (Marks: 4)

(b) Given that $f(x)=\sqrt{x}$ and $g(x)=x^3+1$, find:

(i) $f \circ g$ (2) (ii) $g \circ f$ (4)

(Marks: 5)

(c) Find the expression for $f^{-1}(x)$ for the following function:

$$f(x)=\frac{1+x}{1-x}$$

(Marks: 5)

(d) Find the following limits:

(i)

$$\lim_{x\to-\infty}\frac{\sqrt{3x^4+x}}{x^2-8}$$

(ii)

$$\lim_{t\to 2}\frac{t^3+3t^2-12t+4}{t^3-4t}$$

(Marks: 6)

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Question 2

(a) Solve the following equalities and in each case sketch the solution on a coordinate line:

(i) $3 + 7x \leq 2x - 9$

(ii) $\frac{2x-5}{x-2} < 1$ (Marks: 8)

(b) During the first 40 s of a rocket flight, the rocket is propelled straight up so that in *t* seconds it reaches a height $s(t) = 0.3t^3$ ft.

(i) How high does the rocket travel in 40 s?
(ii) What is the average velocity of the rocket during the first 40 s?
(iii) What is the average velocity of the rocket during the first 1000 ft of its flight?
(iv) What is the instantaneous velocity of the rocket at the end of 40 s?

(Marks: 7)

(c) Find the values of *x*, if any, for which the following functions *f* are not continuous.

(i) $f(x) = 5x^4 - 3x + 7$

(ii) $f(x) = \frac{x+2}{x^2-4}$

(Marks: 5)

Question 3

(a) Find the derivative of the following function:

$$f(x) = \left[\ln\left(\frac{x^2 \sin x}{\sqrt{1+x^2}}\right)\right]$$

(Marks: 7)

(b) First check if the L'Hôpital's rule can be applied to calculate the following limit. If it can be applied then calculate the limiting value of:

$$\lim_{x\to 0} \frac{\sin cx}{x};$$

where c is a constant (Marks: 6)

(c) Assuming that oil spilled from a ruptured tanker spreads in a circular pattern whose area increases at a constant rate of 6 km^2/s. At what rate is the radius of the spill increasing when the area is 8 km^2?

(Marks: 7)

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Question 4

(a) (i) Using the quotient rule, find $\frac{dy}{dx}$ of:

$$y = \frac{(3x^2 + 2)}{x + 3}$$

(ii) Compute the derivative of the following function:

$$f(x) = (2x^7 - x^2)\left(\frac{x-1}{x+1}\right)$$

(Marks: 6)

(b) (i) Evaluate the following definite integral:

$$\int_0^4 (x^3 - 4x - 3)\, dx$$

(ii) Evaluate the following indefinite integral:

$$\text{(ii)} \int \frac{1}{1+\sin\theta} d\theta$$

(Marks: 7)

(c) Find all values of λ for which det(A) = 0:

$$A = \begin{bmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{bmatrix}$$

(Marks: 7)

Question 5

(a) Find the norm of the vector **w**:

$$\mathbf{w} = (-7, 2, -1)$$

(Marks: 4)

(b) Write matrix *A* in the form of $[A \mid I]$, and then by reducing matrix *A* to the form of *I*, find the inverse of *A*.

$$A = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{10} \\ \frac{1}{5} & -\frac{4}{5} & \frac{1}{10} \end{bmatrix}$$

(Marks: 6)

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(c) Find the minor M_{22} and cofactor C_{22} of matrix A given by:

$$A = \begin{bmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{bmatrix}$$

(Marks: 4)

(d) Find a vector that is orthogonal to both vectors **u** and **v** given by:

$$\mathbf{u} = (-6, 4, 2) \text{ and } \mathbf{v} = (3, 1, 5)$$

(Marks: 6)

Question 6

(a) For $\mathbf{u} = (3, 4)$, $\mathbf{v} = (5, -1)$ and $\mathbf{w} = (7, 1)$, calculate:

(i) $\mathbf{u} \bullet (7\mathbf{v} + \mathbf{w})$

(ii) $\|\mathbf{u}\| (\mathbf{v} \bullet \mathbf{w})$

(Marks: 5)

(b) Plot the following points on an Argand diagram.

(i) $2 + 3i$ (ii) $-3 - i$

.

(Marks: 5)

(c) Find all the roots of the following complex number:

$$z = (1 + \sqrt{3}\, i)^{1/2}$$

(Marks: 7)

(d) Express the following complex expression in the form of $a + bi$:

$$i(1 + 7i) - 3i(4 + 2i)$$

(Marks: 3)

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