Charles Darwin University

Final Examination

Family Name	
Given Name/s	
Student Number	
Teaching Period	Semester 1, 2019

SMA209 – Mathematics 2A	DURATION	
	Reading Time:	10 minutes
	Writing Time:	180 minutes

INSTRUCTIONS TO CANDIDATES

1. Answer all **six** questions.
2. All questions are of equal value, and parts carry marks as indicated.
3. Read **ALL** questions carefully.
4. Show all working neatly in all parts. Answers without working details will attract little marks.
5. All symbols, unless stated otherwise, have their usual meanings.

EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
No additional printed material is permitted	1 x 20 Page Book 1 x Scrap Paper Formula Sheet/s

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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Question 1

(a) (i) Write the order of the following ordinary differential equation (ODE):

$$y' + 4y = 1.4$$ **(Marks 1)**

(ii) Verify if the following *y* is the general solution of the above ODE in (i):

$$y = ce^{-4x} + 0.35$$ **(Marks 3)**

(iii) Given the initial value condition $y(0) = 2$, find the particular solution for *y* given in (ii). **(Marks 2)**

(b) Solve the following initial value problem :

$$y' = (1 + x)e^{-x}y^2 \ , y(0) = 1$$ **(Marks 7)**

(c) Find a general solution of the following ODE. Show all steps of derivation neatly.

$$xy' = y + 2x^3\cos^2\left(\frac{y}{x}\right)$$ **(Marks 7)**

Question 2

(a) First check if the following ODE is exact and then find its general solution:

$$\cos(x + y)dx + (3y^2 + 2y + \cos(x + y))dy = 0$$ **(Marks 6)**

(b) Solve the following nonlinear ODE:

$$y' = y - 4y^2$$ **(Marks 7)**

(c) Solve the following second order ODE by reducing it to first order

$$yy'' = 3y'^2$$ **(Marks 7)**

Question 3

(a) Solve the following initial value problems:

(i) $y'' + y' - 6y = 0$, $y(0) = 10$, and $y'(0) = 0$ **(Marks 6)**

(ii) $x^2y'' + 3xy' + 0.75y = 0$, $y(1) = 1$ and $y'(1) = -1.5$ **(Marks 6)**

(b) Find a general solution of the following second order non-homogeneous differential equation:

$$y'' + 2y' + y = e^{-x}$$

(Marks 8)

Question 4

(a) Solve the following initial value problem:

$$y_1' = y_2$$

$$y_2' = y_1 \quad , y_1(0) = 1 \text{ and } y_2(0) = 0$$

(Marks 10)

(b) (i) Sketch the following periodic function by marking the axes clearly:

$$f(x) = \begin{cases} x^2 & -\pi < x < 0 \\ e^x & 0 \le x \le \pi \end{cases}$$

(Marks 5)

$$f(x) = f(x + 2\pi)$$

(iii) Expand the function $f(x)$ in (i) in Fourier series and determine the Fourier coefficient a_0 . **(Marks 5)**

Question 5

(a) If $f(x)$ is a periodic function of x and of period p, show that $f(ax)$ ($a \neq 0$) is a periodic function of x of period p/a. **(Marks 4)**

(b) Sketch the following scalar function by marking the axes clearly:

$$T(x, y) = \sqrt{16 - x^2 - y^2}$$ **(Marks 6)**

(c) The parametric representation of a curve C is given by the vector function **r (t)** as:

$$\mathbf{r}(\mathbf{t}) = [\cos t,\ \sin t, t];\ \ 0 \leq t \leq 2\pi$$

(i) Sketch the curve C clearly by marking the axes. **(Marks 4)**

(ii) Find $\mathbf{r}'(t)$ and then determine the length of the tangent vector at any point t in the interval $0 \leq t \leq \pi$. **(Marks 3)**

(iii) Find the length of the curve C between $t = 0$ and $t = 2\pi$. **(Marks 3)**

Question 6

(a) The elevation $z(x, y)$ of a mountain (in meters) at sea level at a point (x, y) in the xy-plane is given by:

$$z(x, y) = 3000 - x^2 - 9y^2$$

What direction a girl at the point (4, 1) should walk to have the height decrease most rapidly? In what direction will the elevation remain the same? **(Marks 8)**

(b) Find a normal vector of the following surface:

$$6x^2 + 2y^2 + z^2 = 225$$

at the point P: (5,5,5). **(Marks 6)**

(c) (i) Find the divergence of the following vector function**:** **(Marks 3)**

$$\mathbf{v} = xy^2z^2\mathbf{i} + x^2yz^2\mathbf{j} + x^2y^2z\mathbf{k}$$

(ii) Given the vector $\mathbf{F} = xyz(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$, find curl **F.** **(Marks 3)**