Family Name	
Given Names	
Student Number	
Teaching Period	Semester 2, 2016

FINAL EXAMINATION	**DURATION**
ENG223 – Electrical Circuit Analysis	Reading Time: **10** minutes Writing Time: **180** minutes

INSTRUCTIONS TO CANDIDATES

EXAM CONDITIONS

You may begin writing from the commencement of the examination session.

The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
No additional printed material is permitted	1 x 20 Page Book 1 x Scrap Paper

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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QUESTION 1 **(5 Marks)**

The circuit shown in Figure 1 below may be solved in a number of ways including application of KVL and KCL. Of the two Kirchhoff methods, which one would be more economical in terms time and number of steps of calculations? Explain your answer. You don't have to solve the circuit.

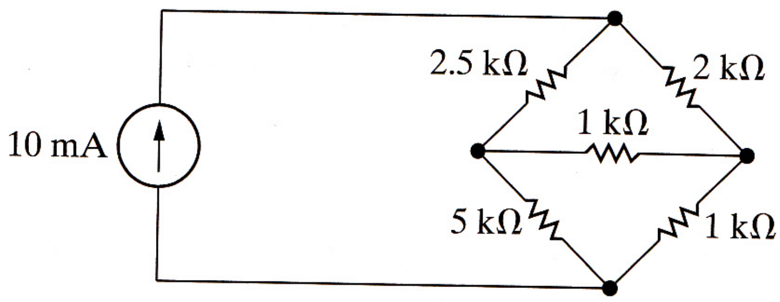

Figure 1

QUESTION 2 **(10 Marks)**

In the circuit shown in Figure 2, the initial capacitor voltage is 15 volts and the initial inductor current is 6 milli amperes.

Q2.1 Determine the value of the resistor R required for critical damping. (3 marks)

Q2.2 Determine the numerical values of current, i, and the rate of change of current di/dt immediately after the switch is closed. (3 marks)

Q2.3 Write an expression for the capacitor voltage as a function of time after the switch is closed. (4 marks)

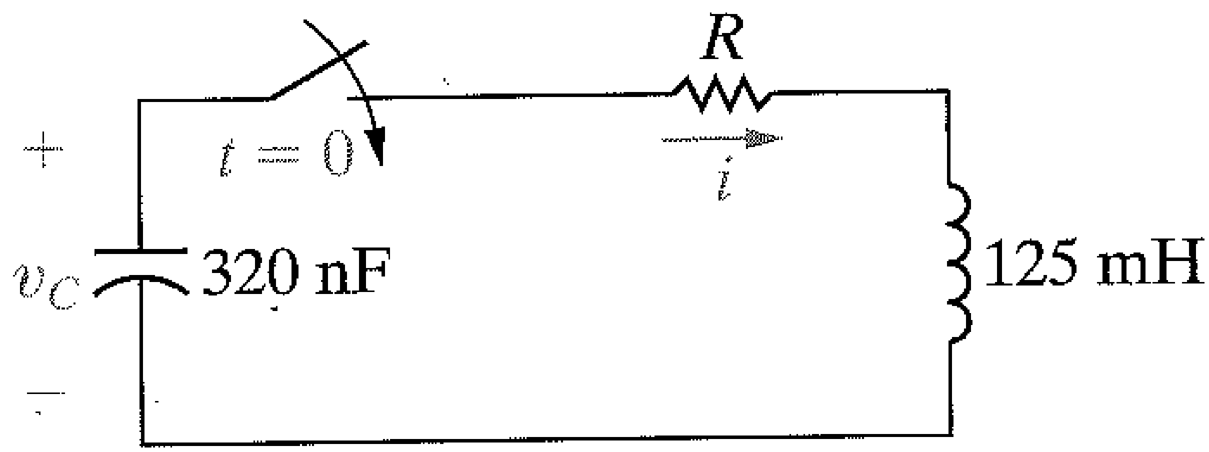

Figure 2

Question 3 **(10 Marks)**

Figure 3 represents an RC high pass filter with an added resistor R_c in series with the capacitor C. (5×2 marks)

Q3.1 Derive the expression for H(s) where H(s) is the ratio, v_o / v_i. Express the result in a simplified form.
Q3.2 Determine the frequency at which H(jω) will have its maximum value.
Q3.3 Determine the maximum value of H(jω).
Q3.4 Determine the frequency at which the magnitude of H(jω) will be 70.71% of its maximum value.
Q3.5 Calculate ω_c, H(j ω_c), H(j 0.125 ω_c), and H(j 8 ω_c).

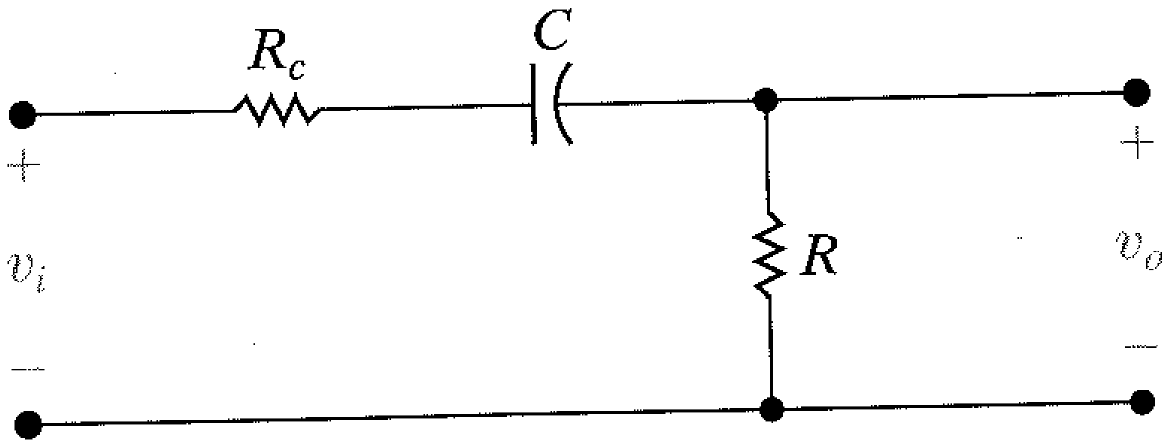

Figure 3

Question 4 **(15 Marks)**

A machine shop has the following three tools. They are all balanced three phase loads and are operated at 220 volts rms. Calculate the total line current these three loads draw from the system.

Load 1: Drill press rated 10.2 kVA at 00.87 lagging power factor.
Load 2: Lathe drawing 4.2 kW at 0.91 lagging power factor.
Load 3: Band saw drawing a line current of 36.8 amps rms and 7.25 kVAr.

Question 5 **(10 Marks)**

One period of a periodic function is described by the following equations:

I(t) = - 8 t amps, $-5 \text{ ms} \le t \le 5 \text{ ms}$
I(t) = - 40 milli-amps, $5 \text{ ms} \le t \le 15 \text{ ms}$
I(t) = 8t – 0.16 amps, $15 \text{ ms} \le t \le 25 \text{ ms}$
I(t) = +40 milli-amps, $25 \text{ ms} \le t \le 35 \text{ ms}$

Q5.1 What is the fundamental frequency of the function in hertz? (1 mark)
Q5.2 Is this function odd or even or neither? (1 mark)
Q5.3 Does the function have half-wave or quarter-wave symmetry? (1 mark)
Q5.4 Determine the Fourier constants a_v, a_k and b_k. (7 marks)

FORMULA SHEET
LAPLACE TRANSFORM PAIRS

F(s)	f(t) $t \ge 0$
1	$u_0(t)$ unit impulse at t = 0
1/s	1 or u(t) unit step starting at t = 0
$1/s^2$	t u(t) ramp function
$1/s^n$	$\frac{1}{(n-1)!}t^{n-1}$ n = positive integer
$\frac{1}{s}e^{-at}$	$u_{-1}(t-a)$ unit step starting at t = a
$\frac{1}{s}\left(1-e^{-at}\right)$	$u_{-1}(t)-u_{-1}(t-a)$ rectangular pulse
1/(s+a)	e^{-at} exponential decay
$1/(s+a)^n$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$ n = positive integer
$\frac{1}{s(s+a)}$	$\frac{1}{a}\left(1-e^{-at}\right)$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[1-\frac{b}{b-a}e^{-at}+\frac{a}{b-a}e^{-bt}\right]$
$\frac{s+\alpha}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[\alpha-\frac{b(\alpha-a)}{b-a}e^{-at}+\frac{a(\alpha-b)}{b-a}e^{-bt}\right]$
$\frac{1}{(s+a)(s+b)}$	$\frac{1}{b-a}\left(e^{-at}-e^{-bt}\right)$
$\frac{s}{(s+a)(s+b)}$	$\frac{1}{a-b}\left(ae^{-at}-be^{-bt}\right)$
$\frac{s+\alpha}{(s+a)(s+b)}$	$\frac{1}{b-a}\left((\alpha-a)e^{-at}-(\alpha-b)e^{-bt}\right)$
$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)}+\frac{e^{-bt}}{(c-b)(a-b)}+\frac{e^{-ct}}{(a-c)(b-c)}$
$\frac{s+\alpha}{(s+a)(s+b)(s+c)}$	$\frac{(\alpha-a)e^{-at}}{(b-a)(c-a)}+\frac{(\alpha-b)e^{-bt}}{(c-b)(a-b)}+\frac{(\alpha-c)e^{-ct}}{(a-c)(b-c)}$

Overdamped

$$\alpha^2 > \omega_0^2$$
$$x(t) = X_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}$$
$$x(0) = X_f + A_1' + A_2'$$
$$dx/dt(0) = A_1' s_1 + A_2' s_2$$

Underdamped

$$\alpha^2 < \omega_0^2$$
$$x(t) = X_f + (B_1' \cos \omega_d t + B_2' \sin \omega_d t) e^{-\alpha t}$$
$$x(0) = X_f + B_1'$$
$$dx/dt(0) = -\alpha B_1' + \omega_d B_2'$$
$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

Critically damped

$$\alpha^2 = \omega_0^2$$
$$x(t) = X_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$$
$$x(0) = X_f + D_2'$$
$$dx/dt(0) = D_1' - \alpha D_2'$$

FOURIER COEFFICIENTS

$$a_v = \frac{1}{T}\int_0^{t_0+T} f(t)dt$$
$$a_k = \frac{2}{T}\int_0^{t_0+T} f(t)\cos(k\,\omega_0 t)\,dt$$
$$b_k = \frac{2}{T}\int_0^{t_0+T} f(t)\sin(k\,\omega_0 t)\,dt$$