Family Name	
Given Names	
Student Number	
Teaching Period	Semester 1, 2017

FINAL EXAMINATION	**DURATION**
ENG325 – Systems Modelling and Control	Reading Time: **10** minutes Writing Time: **180** minutes

INSTRUCTIONS TO CANDIDATES

1.1 The examination has five (5) questions.

1.2 All answers must include a sufficient amount of working and explanation.

1.3 Note that questions ARE NOT of equal value.

1.4 Read ALL questions carefully.

EXAM CONDITIONS

<u>You may begin writing from the commencement of the examination session.</u> The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
none	1 x 20 Page Book 2 x Scrap Paper

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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Question 1 (11 marks)

A robot arm is driven by two actuators as shown below in Figure 1. The actuators can be considered as continuous-time, linear, time-invariant systems with the control signal *x(t)* as the input and the force delivered by the actuators as the output. Actuator 1 has impulse response $h_1(t)$ and actuator 2 has impulse response $h_2(t)$. The forces delivered by the actuators are added together to form the input signal to the robot arm. The output of the robot arm is the position of the tip y(t). The robot arm can also be considered as a continuous time linear time invariant system. It has impulse response $h_3(t)$.

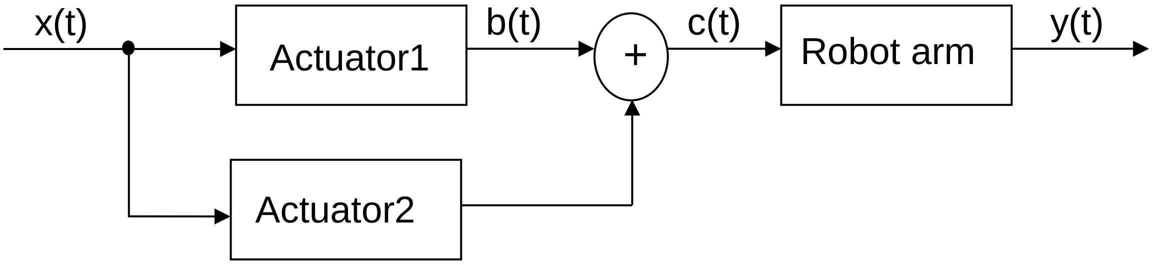

Figure 1. Robot Arm

Question 1.1 (1 marks)

If the input $x(t) = \delta(t)$, where $\delta(t)$ is the unit impulse, what *specifically* is *b(t)*?

Question 1.2 (3 marks)

Give an expression for *y(t)* as a function of *x(t)*, $h_1(t)$, $h_2(t)$ and $h_3(t)$.

Question 1.3 (2 marks)

Write out and expression for the impulse response of the composite system as a function of $h_1(t)$, $h_2(t)$ and $h_3(t)$.

Question 1.4 (2 marks)

The frequency responses of actuator 1, actuator 2 and the robot arm are $H_1(\omega)$, $H_2(\omega)$ and $H_3(\omega)$ respectively. What is the frequency response of the composite system, $H(\omega)$?

Question 1.5 (3 marks)

Is it realistic to model a robot arm as a continuous time, linear, time invariant system? Explain your answer.

Question 2 (12 marks)

The Fourier transform of a signal, $x(t)$, is defined as:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt, \qquad -\infty < \omega < \infty$$

2.1 (4 marks)

Using the above integral, find the Fourier transform of $p_2(t)$: a rectangular pulse shown below in Figure 2(a):

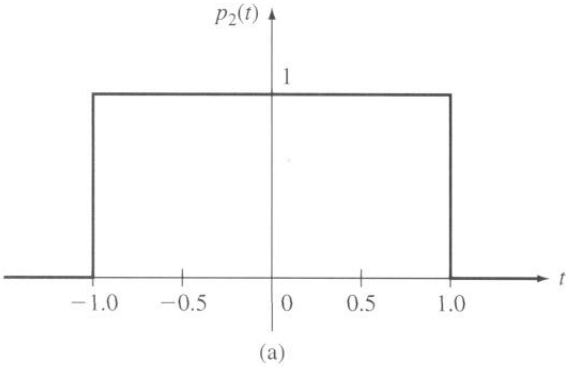

Figure 2

2.2

Once you have the Fourier transform of a particular function, it is possible to find the transform of a more complex function by first expressing the complex function in terms of the simple function and then applying the properties of the Fourier transform.

a) Write an expression in the time domain for the two functions shown below in Figure 3 in terms of the rectangular pulse, $p_2(t)$, shown above in Figure 2. **(4 marks)**

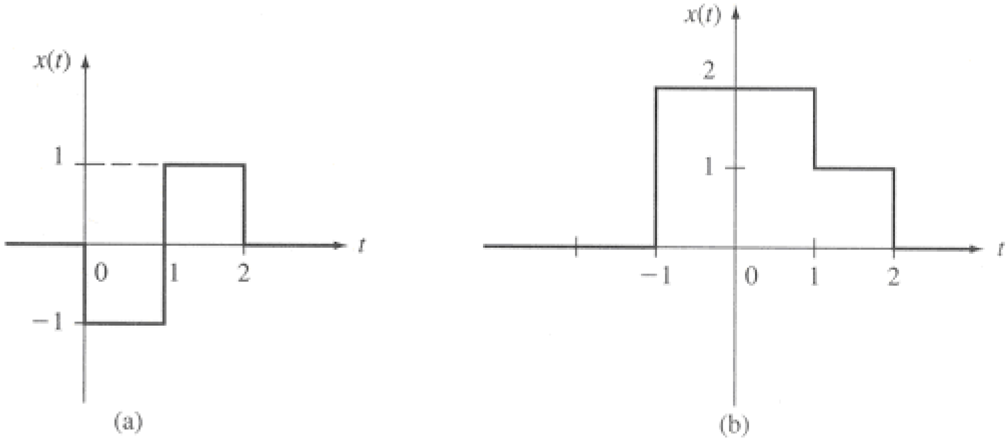

Figure 3

b) Using the properties of the Fourier transform given in the formula sheet attached, find the Fourier transform for the two waveforms in part (a) **(4 marks)**

Question 3 (10 marks)

One way to determine the frequency response, $H(\omega)$, of a given system is to do so experimentally. A number of single frequency sinusoidal signals are used as the input, $x(t)$, to the system, one at a time. The output of the system, $y(t)$, for a given input, can be measured and the magnitude and phase of the frequency response of the system, at the frequency of the input sinusoid, can be measured.

Experimental results for one such test, seen below in Figure 4, show the frequency domain impulse response for the system tested.

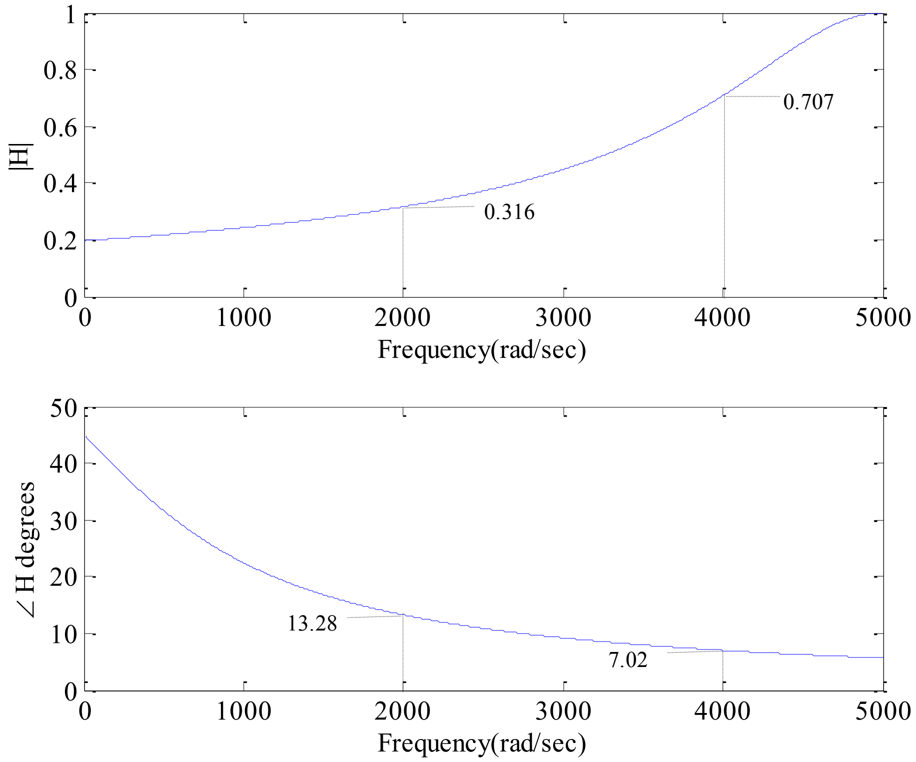

Figure 4.

Question 3.1 (2 Marks)

This system shown in Figure 4 is effectively working as a filter. What type of filter is it? Write a few sentences to justify your answer.

Question 3.2 (4 Marks)

If the input of the system is now: $x(t) = 2\cos(636.6\pi t) + 2\sin(1273.2\pi t)$. Calculate the output of the system, $y(t)$, for this new input. How does your result confirm your choice of filter in question 3.1?

Question 3.3 (4 Marks)

Sketch the double sided frequency domain magnitude and phase plots for both $x(t)$ and $y(t)$. (Present your answer in 4 separate plots, not all on one plot.)

Question 4 (17 marks)

A student wants to relax after an exam and decides to have a drink with his friends. He finishes his first beer, which contains 10 grams of alcohol, after 10 minutes.

Question 4.1 (2 marks)
If the student continues to drink at the same rate what is the Laplace transform of his alcohol ingestion rate $x(t)$? Assume that the student starts drinking at $t=0$ and the alcohol ingestion rate can be approximated as being constant.

Question 4.2 (3 marks)
Determine the transfer function of system described by the equations for the ingestion and metabolism of alcohol given below:

The ingestion and metabolism of alcohol described by the following equations

$$\frac{dq(t)}{dt} = -k_1 q(t) + x(t) \quad and \quad \frac{dy(t)}{dt} = k_1 q(t) - k_2 y(t)$$

where the input $x(t)$ is the ingestion rate of alcohol, the output $y(t)$ is the total amount of the alcohol in the bloodstream and $q(t)$ is the total amount of the alcohol in the gastrointestinal tract. The constants k_1 and k_2 characterize the speed with which alcohol is absorbed from the gastro-intestinal tract into the bloodstream and the speed with which alcohol is metabolized by the liver respectively. It is assumed that $k_1 > k_2 > 0$. (For this question only you can assume that initial values are zero.

If and only if you cannot find the transfer function, use the following transfer function for question 4.3 – 4.7.

$$H(s) = \frac{k_1 k_2}{(s + k_1)(s + k_2)}$$

Question 4.3 (1 marks)
What is the order of the system? Explain your answer, don't just state a number.

Question 4.4 (2 marks)
What is the location of the poles of the system?

Question 4.5 (2 marks)
Is the system stable? Explain your answer.

Question 4.6 (3 marks)
Determine the impulse response $h(t)$ of the system.

Question 4.7 (4 marks)
Finally the student stops drinking and decides to go home. At that time, the amount of alcohol in his gastro-intestinal system is M_1 and the amount of alcohol in his blood is M_2. By using the Laplace transform, compute the amount of alcohol in the students blood, $y(t)$, from the moment he stops drinking onward. Assume that $k_1 \neq k_2$.

Question 5 (15 marks)

Question 5.1 (2 marks)

A controller with transfer function $G_C(s)$ is used to provide the input into a plant with transfer function $G_P(s)$. The input to the controlled system is the reference signal $R(s)$which produces the output, $Y(s)$, from the plant. Draw the block diagram for the system described if:

a) open loop control is used
b) closed loop control is used

Question 5.2 (2 marks)

Give an example of an everyday system where open loop control is used and another where closed loop control is used.

Question 5.3 (5 marks)

What are the reasons that closed loop control is sometimes used instead of open loop control. What are some of the advantages and disadvantages of both open loop and closed loop control?

Question 5.4 (6 marks)

Discuss, with the use of formula and diagrams where relevant, how you would use one of the two tuning methods discussed in this unit to tune a PI controller.

Formula Sheet

LAPLACE TRANSFORM PAIRS

$F(s)$	$f(t) \quad t \geq 0$
1	$\delta(t)$ unit impulse at t = 0
$\frac{1}{s}$	1 or $u(t)$ unit step starting at t = 0
$\frac{1}{s^2}$	$tu(t)$ ramp function
$\frac{1}{s^n}$	$\frac{1}{(n-1)!}t^{n-1}$ n = positive integer
$\frac{1}{s}e^{-at}$	$u\ (t-a)$ unit step starting at t = a
$\frac{1}{s}\left(1-e^{-at}\right)$	$u\ (t)-u\ (t-a)$ rectangular pulse
$\frac{1}{s+a}$	e^{-at} exponential decay
$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$ n = positive integer
$\frac{1}{s(s+a)}$	$\frac{1}{a}\left(1-e^{-at}\right)$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[1-\frac{b}{b-a}e^{-at}+\frac{a}{b-a}e^{-bt}\right]$
$\frac{s+\alpha}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[\alpha-\frac{b(\alpha-a)}{b-a}e^{-at}+\frac{a(\alpha-b)}{b-a}e^{-bt}\right]$
$\frac{1}{(s+a)(s+b)}$	$\frac{1}{b-a}\left(e^{-at}-e^{-bt}\right)$
$\frac{s}{(s+a)(s+b)}$	$\frac{1}{a-b}\left(ae^{-at}-be^{-bt}\right)$

$\frac{s+\alpha}{(s+a)(s+b)}$	$\frac{1}{b-a}\left((\alpha-a)e^{-at}-(\alpha-b)e^{-bt}\right)$
$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)}+\frac{e^{-bt}}{(c-b)(a-b)}+\frac{e^{-ct}}{(a-c)(b-c)}$
$\frac{s+\alpha}{(s+a)(s+b)(s+c)}$	$\frac{(\alpha-a)e^{-at}}{(b-a)(c-a)}+\frac{(\alpha-b)e^{-bt}}{(c-b)(a-b)}+\frac{(\alpha-c)e^{-ct}}{(a-c)(b-c)}$
$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$
$\frac{s}{s^2+\omega^2}$	$\cos\omega t$
$\frac{s+\alpha}{s^2+\omega^2}$	$\frac{\sqrt{\alpha^2+\omega^2}}{\omega}\sin(\omega t+\theta)$ $\phi=\tan^{-1}\left(\frac{\omega}{\alpha}\right)$
$\frac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$	$\sin(\omega t+\theta)$
$\frac{1}{s(s^2+\omega^2)}$	$\frac{1}{\omega^2}(1-\cos\omega t)$
$\frac{s+\alpha}{s(s^2+\omega^2)}$	$\frac{\alpha}{\omega^2}-\frac{\sqrt{\alpha^2+\omega^2}}{\omega^2}\cos(\omega t+\phi)$ $\phi=\tan^{-1}\frac{\omega}{\alpha}$
$\frac{1}{(s+a)(s^2+\omega^2)}$	$\frac{e^{-at}}{a^2+\omega^2}+\frac{1}{\omega\sqrt{\alpha^2+\omega^2}}\sin(\omega t-\phi)$ $\phi=\tan^{-1}\frac{\omega}{a}$
$\frac{1}{(s+a)^2+b^2}$	$\frac{1}{b}e^{-at}\sin bt$
$\frac{1}{s^2+2\xi\omega_n s+\omega_n^2}$	$\frac{1}{\omega_n\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\omega_m\sqrt{1-\xi^2}\,t$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
$\frac{s+\alpha}{(s+a)^2+b^2}$	$\frac{\sqrt{(\alpha-a)^2+b^2}}{b}e^{-\alpha t}\sin(bt+\phi)$ $\phi=\tan^{-1}\frac{b}{\alpha-a}$

$\frac{1}{s\left[(s+a)^2+b^2\right]}$	$\frac{1}{a^2+b^2}+\frac{1}{b\sqrt{a^2+b^2}}e^{-\alpha t}\sin(bt-\phi)$ $\phi=\tan^{-1}\frac{b}{-a}$
$\frac{1}{s\left(s^2+2\xi\omega_n s+\omega_n^2\right)}$	$\frac{1}{\omega_n}-\frac{1}{\omega_n^2\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\left(\omega_n\sqrt{1-\xi^2}t+\phi\right)$ $\phi=\cos^{-1}\xi$
$\frac{s+\alpha}{s\left[(s+a)^2+b^2\right]}$	$\frac{\alpha}{a^2+b^2}+\frac{1}{b}\sqrt{\frac{(\alpha-a)^2+b^2}{a^2+b^2}}e^{-\alpha t}\sin(bt+\phi)$ $\phi=\tan^{-1}\frac{b}{c-a}-\tan$
$\frac{1}{(s+c)\left[(s+a)^2+b^2\right]}$	$\frac{e^{-ct}}{(c-a)^2+b^2}+\frac{e^{-at}\sin(bt-\phi)}{b\sqrt{(c-a)^2+b^2}}$ $\phi=\tan^{-1}\frac{b}{c-a}$

Final Value Theorem for Laplace transform pairs:

$$\lim_{t\to\infty}\theta(t)=\lim_{s\to 0}s\theta(s)$$

Ziegler-Nichols tuning rule based on critical gain and period

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$
$$= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right)$$
$$= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s}$$

Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Ziegler-Nichols tuning rule based on the strep response of the system:

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$
$$= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right)$$
$$= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s}$$

Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

Properties of the Fourier Transform

Property Name	Property	
Linearity	$ax(t)+bv(t)$	$aX(\omega)+bV(\omega)$
Time Shift	$x(t-c)$	$e^{-j\omega c}X(\omega)$
Time Scaling	$x(at), \quad a \neq 0$	$\frac{1}{a}X(\omega/a), \quad a \neq 0$
Time Reversal	$x(-t)$	$X(-\omega)$ $\overline{X(\omega)}$ if $x(t)$ is real
Multiply by t^n	$t^n x(t), \quad n=1,2,3,\ldots$	$j^n \frac{d^n}{d\omega^n}X(\omega), \quad n=1,2,3,\ldots$
Multiply by Complex Exponential	$e^{j\omega_o t}x(t), \quad \omega_o$ real	$X(\omega-\omega_o), \quad \omega_o$ real
Multiply by Sine	$\sin(\omega_o t)x(t)$	$\frac{j}{2}[X(\omega+\omega_o)-X(\omega-\omega_o)]$
Multiply by Cosine	$\cos(\omega_o t)x(t)$	$\frac{1}{2}[X(\omega+\omega_o)+X(\omega-\omega_o)]$
Time Differentiation	$\frac{d^n}{dt^n}x(t), \quad n=1,2,3,\ldots$	$(j\omega)^n X(\omega), \quad n=1,2,3,\ldots$
Time Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda$	$\frac{1}{j\omega}X(\omega)+\pi X(0)\delta(\omega)$
Convolution in Time	$x(t)*h(t)$	$X(\omega)H(\omega)$
Multiplication in Time	$x(t)w(t)$	$\frac{1}{2\pi}X(\omega)*W(\omega)$
Parseval's Theorem (General)	$\int_{-\infty}^{\infty} x(t)\overline{v(t)}dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\overline{V(\omega)}d\omega$	
Parseval's Theorem (Energy)	$\int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$ if $x(t)$ is real $\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$	
Duality: If $x(t) \leftrightarrow X(\omega)$	$X(t)$	$2\pi x(-\omega)$

Properties of Discrete time Fourier transform

Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
Shift in Time	$x[n-k]$	$X(e^{j\omega})e^{-j\omega k}$
Shift in frequency	$x[n]e^{jan}$	$X(e^{j(w-a)})$
Time scaling	$x\left[\frac{n}{k}\right]$	$X(e^{j(k\omega)})$
Time reversal	$x[-n]$	$X(e^{-j\omega})$
Time conjugation	$x[n]^*$	$X(e^{-j\omega})^*$
Time reversal and conjugation	$x[-n]^*$	$X(e^{j\omega})^*$
Derivative in frequency	$\frac{n}{j}x[n]$	$\frac{dX(e^{j\omega})}{d\omega}$
Integral in frequency	$\frac{j}{n}x[n]$	$\int_{-\pi}^{\omega} X(e^{jv})dv$
Convolve in time	$x[n] * y[n]$	$X(e^{j\omega}).Y(e^{j\omega})$
Multiply in time	$x[n].y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jv}).Y(e^{j(\omega-v)})dv$